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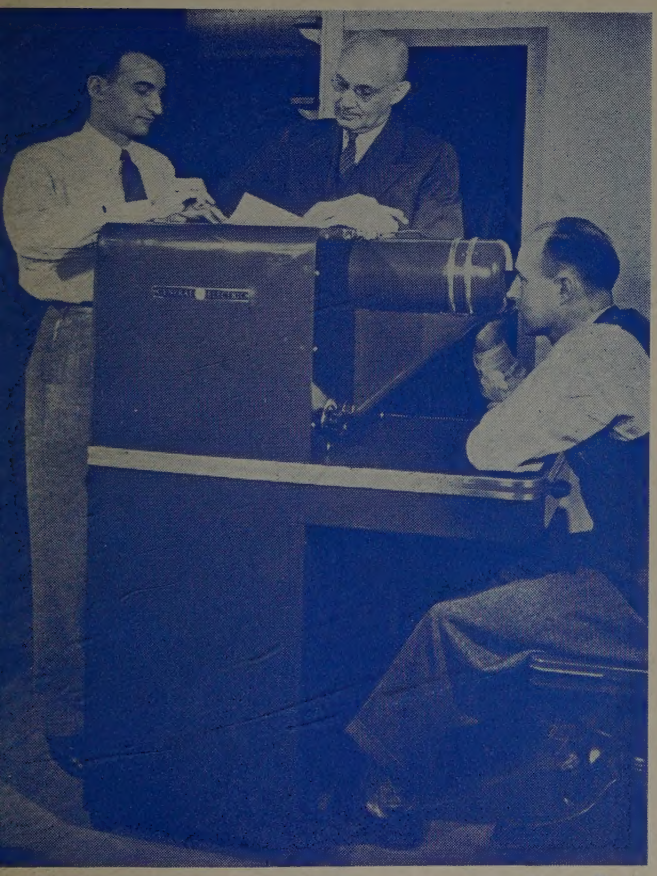

General Electric

RADIO-AND-ELECTRONIC ENGINEERS
I.R.E. Members Checking Operation of Their Electron Microscope

AUGUST, 1944

VOLUME 32 NUMBER 8

Centennial of the Telegraph

Electronic Heaters

Instrument to Measure pH

Z-Marker Antenna System

Vacuum Capacitors

Voltage-Multiplying Rectifiers

Velocity-Modulation Tubes

Electron-Image Formation

Institute of Radio Engineers

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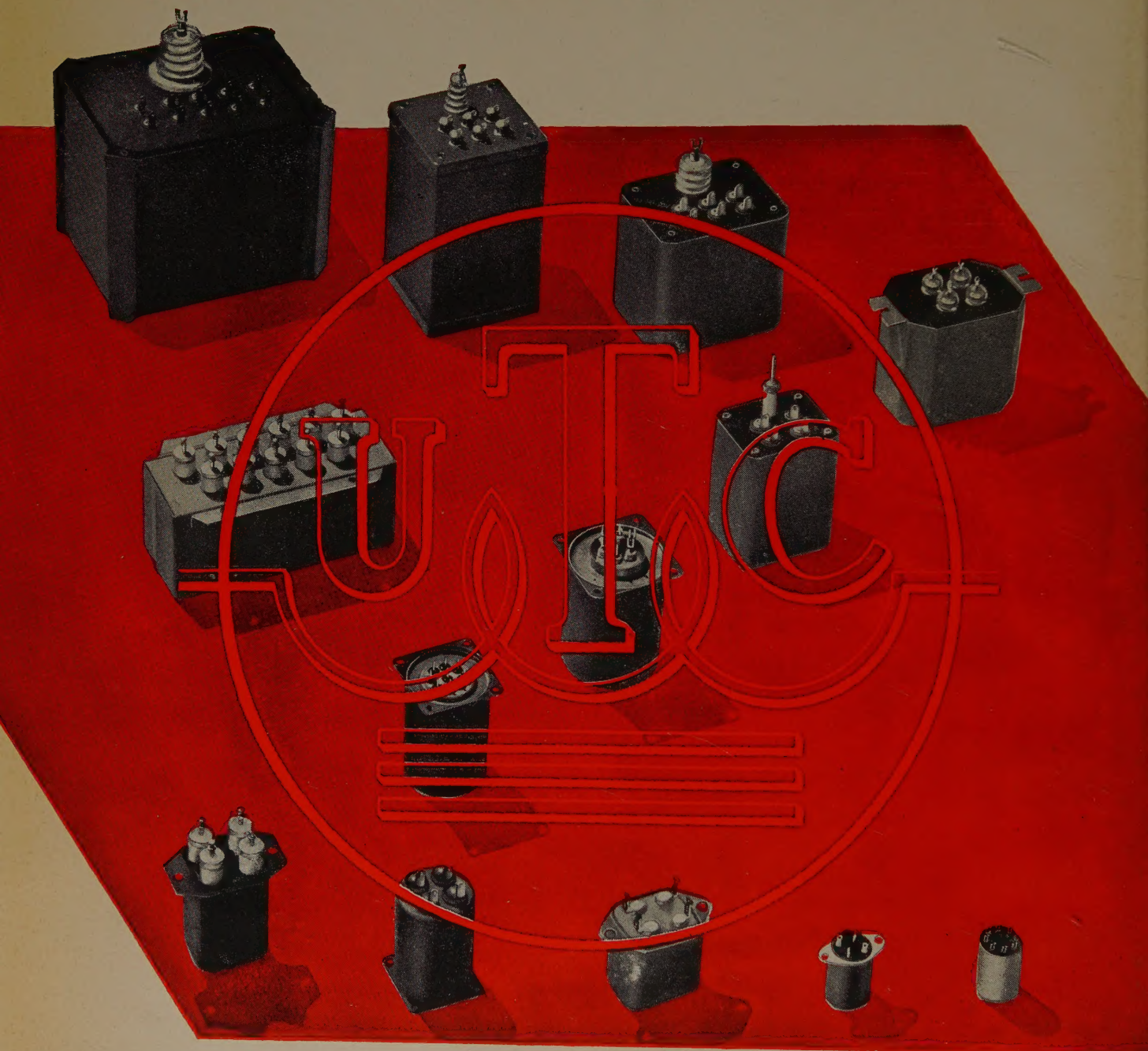

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Close co-operation and understanding between the radio-and-electronic engineers, and industrial pioneers and leaders in their industry will undoubtedly promote not only personal good will but also technical, individual, and organizational effectiveness. To these ends, the PROCEEDINGS OF THE I.R.E. presents in the form in which they are received expressions of the viewpoints of the outstanding figures in this field. The thoughts expressed below by the President of the Mutual Broadcasting System are a contribution to this interchange of ideas between the engineers and their industrial colleagues.

The Editor

The Radio Engineer's Stake in Our Future

MILLER McCLINTOCK

One of the most inspiring elements in broadcasting lies not only in the breath-taking speed with which it has developed but likewise in the fact that the horizons ahead have no limitation.

The miracle of radio as it stands today is the result of the imagination, skill, and scientific ability of radio engineers in all fields of electronics. Upon these engineers, we must continue to depend for the expansion of radio to its full opportunities and responsibilities in the future.

That which was an unbelievable phenomenon only a few years ago has now become an actuality. It is a simple commonplace to say that frequency modulation and television will be here in great volume in the postwar years. While its full development cannot yet be foreseen, it needs little discussion among radio engineers.

We are less likely, however, to understand the impact of the radio industry upon the development of other industries and upon the molding of our social and economic life.

Immediately after the war and in the years following, automobiles will return in full volume. There may be as many as 50,000,000 cars in this country ten years after peace. The principles are already laid whereby the highways of the future and the traffic system will largely be under electronic control. Each vehicle will be equipped with radio signals on its instrument panel telling, among other things, whether or not cars are coming from blind roadways or drives ahead. Traffic stop and go signals will be repeated in cars both by light and by sound. Continuous traffic instructions and directions will be available along each highway for each car. Touring passengers may enjoy a description of the historical and business significance of each of the miles of the highway over which they pass. These mechanisms are not visionary but have been fully field-tested and probably no highway will be built in the future without complete electronic equipment.

There will be many electronic controls within cars themselves, such as electric bumpers preventing collisions and invisible electric tracks to guide vehicles around curves and obstacles. Passengers in cars will find it possible, at 60 miles per hour, to call any telephone number in the country. The effects of these developments upon comfort, safety, and efficiency in highway traffic are beyond calculation.

The application of radio to maritime and aerial navigation opens equally practical but, none the less, spectacular opportunities. Shipping upon the approaches to and in great harbors will no longer be fogbound. Ships will be guided to their berths and along their channels with complete security and efficiency. Collisions at sea will become something of history.

Air liners similarly will descend unseeing in heavy weather to land with safety on their home runways. But to return to land transportation; radio as developed today has all of the elements necessary to make the tragic train disasters of recent months only a memory. Full and complete radio train control is now available in several different forms. Engineers will never be without full information as to the clear track ahead and should they disregard warning control, the train control will be taken away from them.

These are only a few of the marvels of radio which will be commonplace things of tomorrow. If one goes into the laboratories, the factory, the great steel mills, he only sees today the elementary beginnings of electronic controls of scientific processes and production which will revolutionize many industrial activities.

All of these developments will be superimposed upon the greatly expanding services and obligations of radio communications for the entertainment, information, and culture of our people and of the people of the whole world. In the hands of the radio engineer lies the opportunity and the capacity to make all of these marvels the servants of mankind.

Fabian Bachrach

E. Finley Carter

Board of Directors—1944

E. Finley Carter was born in Elgin, Texas, on July 1, 1901. He was graduated with distinction, receiving a B.S. degree in electrical engineering from Rice Institute in 1922. Mr. Carter started work for the General Eletric Company in radio development at Schenectady in 1922. One of his earliest jobs was the development of the three early high-power broadcast transmitters, WGY, Schenectady; KGO, Oakland; and KOA, Denver. He held various positions in charge of developmental sections and at the time of his leaving was division engineer in charge of a special development division handling television, facsimile, and commercial receiver developments. In 1926, he spent three months installing the first General Electric carrier-current equipment involving the use of coupling condensers on 220-kilovolt lines, Southern California Edison and Pacific Gas and Electric systems, from Los Angeles to the High Sierras and from Oakland to the Pit River stations. He traveled extensively throughout the country in development of carrier-communications systems.

On May 1, 1929, Mr. Carter became director of the radio division of the United Research Corporation, New York, a subsidiary of Warner Pictures, designing Brunswick radios, circuits, and receivers. He holds a number of United States and foreign patents assigned to General Electric, United Research, and Sylvania Electric Products, on devices ranging from electronic-control devices to single-frequency duplex transmission and reception systems, radio-receiver systems, and vacuum tubes.

In September, 1932, he joined Sylvania Electric Products, Inc., as a consulting engineer, later becoming assistant chief engineer. In February, 1941, he was appointed to organize and head a new industrial relations department of Sylvania, taking the position of director of industrial relations. He was elected a director of the company in March, 1943.

Mr. Carter is an Associate member of the American Institute of Electrical Engineers, a member of the American Radio Relay League, and Tau Beta Pi. He joined the Institute of Radio Engineers as a Fellow in 1936, and is now a member of the Board of Directors.

A Critique of Communication at the Centennial of the Telegraph*

I. S. COGGESHALL†, FELLOW, I. R. E.

Summary—**The historical influence of S. F. B. Morse, American inventor of the telegraph, upon present-day life is attributed to the fundamental soundness of his work, to his being first in the field of applied electricity, and to his recognition of the social forces reacting to his invention. He is credited with having inspired a technological following who laid the foundations of electrical and radio engineering and education. The paper appraises the effect of electrical communication upon world peace, and poses the question of engineering responsibility in the postwar world.**

IT IS A felicitous circumstance that the New York Section of The Institute of Radio Engineers should have chosen this auditorium of the "Telegraph Capitol of the World" as the place in which to hold a conference on industrial electronics this particular evening, for it was one hundred years ago to the day, May 24, 1844, that Morse sent his epochal "What Hath God Wrought!" message by telegraph from the Capitol at Washington to Baltimore.

Applied electricity on that memorable day a century ago figuratively flashed its way out of the physics laboratory into the globe-encircling field of its later triumphs. Hitherto electricity functioned only in the experiments and classroom demonstrations of Gilbert, Franklin, Volta, Davy, Ampere, Ohm, Faraday, and Henry; thereafter there was to be a continuous stream of instances where the products of electrical experiment were to be applied to the workaday world. This centennial is therefore quite the first of a long succession which will celebrate, in turn, the other electrical inventions, including those of radio and electronics, which have combined to revolutionize power, illumination, manufacturing, transportation, and communication, and hence the very pattern and mode of modern living.

I

Morse's competition was the United States mail, which by that time was being moved by railroad, and the semaphore, a visual fair-weather telegraph relay system which had grown up in this country after having served the Rothschilds and others well in the Napoleonic Wars. The semaphore's tariffs gave Morse an idea of what he might expect, both for patronage and revenues, as the result of public acceptance of his invention, and he solicited the backing of the Congress to build an experimental line. The five years' opposition which he met was due to the initial unwillingness of that body to accept the risk of the not-understood electrical device against the demonstrated utility and obviousness of the alternative visual semaphore.

The alphabetic code which Morse devised was a triumph of simplicity itself. In it he set up a relationship of language, alphabetically expressed, and of electric impulses, measured in time, which not only stood the test of immediate acceptance but has met, in 100 years, only one serious competitor, the five-unit code adapted from Baudot. It is a tribute to the inventor that even today his code (with a minor variation introduced on the European continent in 1851, from which it derives the name of Continental Morse), carries all but a fraction of the world's radiotelegraph traffic. The submarine-cable adaptation of the code, using positive, negative, and zero-current values, and averaging only 3.7 impulses per transmitted character, still sets the practical record for economy of line time in the transmission of thought by electricity. It is evident that the significance of the ETAOIN SHRDLU arrangement of the modern linotype keyboard was not lost on Morse even as far back as 1844. His code was fundamentally sound.

Morse and his partner, Alfred Vail, were their own first operators, the first of the amateurs, the first professional telegraphers, the first of three generations or more of ambitious American youth to whom the telegraph and the railroad offered early hobbies and life work the equivalent of today's radio and aviation. The contrast between Morse's poverty during the interval of his life when he was obsessed with his invention, and his prior success as an artist, and his eventual success as entrepreneur of the industry which he founded, fitted nicely into the picture of the times. He was an individualist, quite rugged, an exponent of competitive enterprise, an indefatigable defender of his patent rights, and a good businessman.

His influence upon Cyrus Field, who first joined Europe and America telegraphically, was direct and personal. Perhaps even more important for posterity, Morse fired the imagination of the youthful Edison and by that one circumstance alone catalyzed the electrical arts. Alexander Graham Bell was in quest of a harmonic telegraph when he stumbled upon the principle of electrical transmission of speech and invented the telephone. The telegraph also was the early objective of Marconi, who sought a means for, and succeeded in, transmitting Morse code between mobile stations without wires; and of de Forest, whose vacuum tube, having already revivified communication, promises to revitalize the use of power through industrial electronics.

Morse lived his life within the narrow portion of the

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† General Cable Supervisor, Western Union Telegraph Company, New York, N. Y.

electrical-communication spectrum lying between zero and 15 cycles per second and quite without the benefit of electrical engineering, for his art was itself the progenitor of electrical engineering. Electrical and magnetic phenomena were originally the province of natural science, or physics, and it was in the physics laboratory of New York University that Morse worked out his dream of 1832 and improved his original instruments to such an extent before patenting them in 1840 that it may be said that the telegraph was not a garret invention but one produced under college auspices. Morse also received help on a university level from Henry's experiments with powerful electromagnets at Princeton, and scientific support from the endorsement of his work by the Franklin Institute in Philadelphia.

One of the first problems to be solved in the field was the matching of impedances of line and instruments for maximum energy transfer. While this was at first done by methods of cut and try, it was not long before a science grew up around the telegraph, and before its characteristics were made the subject of the first practical electrical measurements. Following the galvanometer and the Wheatstone bridge, came Sir William Thomson, Lord Kelvin, who, in connection with the Atlantic cable, developed the laws of the flow of electricity along a wire having resistance and capacitive reactance, by analogy from the mathematical laws of the flow of heat along conductors. Kirchhoff derived the "telegraphic equation," relating propagation to the electrical parameters of a circuit including inductive reactance and leakance. Contemporaneously with the mathematicians and the physicists came the inventors, with their multiple-channel circuits, their tickers and printers, their improved batteries, and finally steam-engine-driven dynamos for telegraph power. The telegraph electricians began to meet and exchange views. In Great Britain, the Society of Telegraph Engineers underwent metamorphosis into the present Institution of Electrical Engineers. In the United States, 21 telegraph men and four others joined to call into existence the American Institute of Electrical Engineers, the first two presidents of which were telegraph men. Many of the electrical engineering departments of today's universities were founded by men who themselves studied electricity in college physics because of their early interest in telegraphy. When Andrew Carnegie, himself a Morse telegrapher, gave the Founder engineering societies their present headquarters building in New York, he said, with satisfaction: "Well! the telegraph boys will now have a comfortable place to meet." By these various lines of genealogy do we electrical engineers, radio engineers, industrial electronics engineers, trace our ancestry back to Samuel Finley Breese Morse.

II

For 25 years following 1844, the telegraph occupied the field of industrial electricity alone. During that period and in succeeding years, it galvanized the world into reacting at a stepped-up tempo which, to historians, is the century's outstanding characteristic.

Not all philosophers are in agreement that facility of communication, especially under electrical stimulus, is an unmixed blessing. It is interesting to note, for example, that Morse's instrument had been writing its messages for only a year when Thoreau went to Walden Pond to think:

"Our inventions," said he, "are but improved means to an unimproved end, an end which it was already but too easy to arrive at. We are in great haste to construct a magnetic telegraph from Maine to Texas; but Maine and Texas, it may be, have nothing important to communicate. Either is in such a predicament as the man who was earnest to be introduced to a distinguished deaf woman, but when he was presented, and one end of her ear trumpet was put into his hand, had nothing to say. Men think that it is essential that the nation have commerce and talk through a telegraph and ride 30 miles an hour. For my part, I could easily do even without the post office. I think that there are very few important communications made through it. I never received more than one or two letters in my life that were worth the postage and I am sure that I never read any memorable news in a newspaper. To a philosopher all news, as it is called, is gossip, and they who edit it and read it are old women over their tea."

By that standard, five generations of communication men since Morse have been wasting their time! Still, the mailbags do bulge, and the printing presses roar; the telegraph extends many miles beyond Maine and Texas; a thousand broadcast stations emit ten thousand news bulletins a day; and the telephone, in all candor, seldom finds us speechless. As nature is profligate, so are our communication facilities. We feel we are philosophical when we merely concede that nine tenths of them are wasted (at the usual tariffs), and hope that the remainder are of tremendous importance.

Morse figuratively anchored his first telegraph line to a printing press, for unerringly he sensed from the beginning that its greatest social influence would be in the gathering and dissemination of news. In fact, in the early days, telegraph personnel acted as news collectors, or reporters. In time that was outgrown, but the telegraph has never outgrown its importance in keeping the public informed of events as they occur. While newspapers and broadcasting are the modern agencies, both are constantly fed by wire, cable, and radiotelegraph. Today, Freedom of Access to Information is looked upon as one of the peacetime blessings of constitutional government, equal, in the scale of human liberties, to Freedom of Speech guaranteed by the Bill of Rights.

Likewise unerringly, Morse ran his first telegraph along a railroad right of way. Transportation and communication have always been the handmaids of commerce. The telegraph-railroad alliance was a natural one. The railroads needed the telegraph to dispatch their

trains, often single-track-operated in those days, with elements of safety of life as important as elements of speed. Pole lines were most easily erected along railroad rights of way, and since the railroad station was the center of community social life as well as of trade and transportation, the telegraph was in the right place when it was in the "depot." Shoulder to shoulder in this alliance, the railroad and the telegraph literally rolled back the western frontiers of the United States to the Pacific Coast; they opened a continent to exploitation and settlement, and thereby aided in making this country great.

Politically, the telegraph made it possible to administer as a federal unit so vast a territory as that between the Atlantic and Pacific. Abroad, it accomplished somewhat the same end, knitting scattered provinces together and encouraging a growing sense of nationalism throughout the world. In the form of the submarine cable, the telegraph made feasible the administration of the globe-encircling British Empire from the Mother Country. To a certain extent, therefore, Morse's invention determined some of the important world political alignments which have existed down to the present day.

Another social effect of the telegraph was its democratization of the means by which the common man extends his horizons by indulging his taste for movement and adventure. The telegraph became, like the railroad, a "common carrier," serving poor and rich alike with equal favor. Between cities, the horse-driven vehicle and the courier had been the agencies of the wealthy and of the aristocracy. While universal mail service had broken down that barrier, following the diffusion of knowledge through mass education, the semaphore had grown up as a tool of the moneyed interests, a device to get closely guarded market and shipping news through to the big merchant quickly so that he could act to his advantage before others had the reports. Morse promptly broke that tradition down with his electric telegraph, making it available to all the people, to Big Business and Little Business, alike. The railroads did their part by carrying passengers at low rates and without discrimination. The influence of these two factors can hardly be overevaluated. They made the population mobile enough to take advantage of the huge natural resources of the new republic, and on a soundly democratic basis; they conditioned the people upward to a better standard of living, to a new-found independence of spirit, and to a rapid leveling-off of the class system under which their forebears had lived and done business in Europe.

III

Nevertheless, after giving the telegraph due credit for all its social benefits, it is a matter of common observation that it has not brought the millenium. At the time when Field stretched Morse's telegraph across the Atlantic, people generally thought that it might. It was predicted that the establishment of electrical communication between the nations would so merge their spheres of interest as to diffuse civilization, liberty, and law throughout the world. The poets burst forth with such paeons as this:

> "'Tis done! The angry sea consents.
> The nations stand no more apart:
> With claspèd hands the continents
> Feel throbbings of each other's heart."

The poets, in their passion, failed to take into account the relative influence of the electrical as compared to the other important agencies of international communication, notably the physical transportation of men and books, which was going on long before the times of Morse, Field, Bell, or Marconi. Electricity, it is true, emancipated communication from the deadweight of transportation, and electrical communication was thereafter destined to supply nervous energy to the international body politic. But the real vitality continued to lie deep in the accumulated wisdom of the ages, recorded in books, in works of art, and other records of man's progress and culture. The annals of science and engineering, and of that realm of current thought which interprets the plain folk of one country to those of another, continued to be transmitted by mail in the more serious periodicals, our own PROCEEDINGS OF THE I. R. E. being a not inconspicuous example of a publication having an international following. Again, understanding among the nations, and misunderstanding, for that matter, are still fostered by travelers: traders, artists, diplomats, educators, students, tourists. The good they accomplish, and the damage they do, have been, up to now, quite comparable in magnitude to the best and the worst to be imputed to cable and wireless.

The transmission of international thought in pure and undistorted form is a fearsome thing to contemplate, even though the medium be a stretch of passive copper, or the virgin reaches between the sea and the reflecting canopies of the sky. The problems still are there when the radio or cable engineer has compensated for all elements of attenuation and phase to Fourier's complete satisfaction, after he has canceled out his last parasitic stray, and lifted his gain by the last useful decibel. For electrical communication shares with all forms of the written and spoken word the dangers of unintentional distortion through all the vagaries known to semantics. In addition, when the complex of telegraphic brevity, or the exigencies of radio program-making, interfere, ideas are sometimes only half-expressed, leading to difficulties not usually attributed to the medium of transmission. Furthermore, the very celerity with which remarks can be interchanged between nations too frequently leads to hair-trigger promptness and asperity of rejoinder. Conjure those three elements and you may witness one of those characteristic transoceanic exhibitions of editorial and parliamentary shuttlecock which so often put international good will to a temporary strain. Even the accompaniment of a parochial accent, or an unfamiliar turn of expression, may greatly

aggravate the seizure; the absence of a common language has been known to prove fatal.

During a state of war, further distortions are introduced intentionally, for military reasons. Fact and propaganda, it has been found, are equally easily translated into the long and short impulses of the telegraph codes. Words can be conveniently twisted into ambiguity. Only a thin veil separates the missions of news to inform in times of peace and to influence in time of war, and the veil is gently dropped as the transition takes place. Almost without being aware of it, one becomes inured to the truth, the half-truth, and anything but the truth.

IV

Disillusioned though we, of the freemasonry of engineering and science, may feel, and chagrined at the failure of half the world's leadership to employ the agency of worldwide facilities of communication to achieve enduring peace rather than to precipitate a technologically cunning but devastating and exhaustive war, nevertheless the future is inexorably being forged to impel a return to sanity on a global scale. The long-term thesis still seems incontrovertible that improvement of the quality of communication between the nations eventually will not fail to aid all mankind to find common interests and to pursue them in harmony. We may still look forward to that day when the interchange of thought between the citizens of all the world shall be so continuous, so untrammeled, and so intimate, that there will in fact be produced common denominators of high purpose, catholicity of interest, a human sympathy far wider than national borders or mother tongue, and genuine tolerance of differentiation.

Surely, the communication engineer of the future, in harnessing electricity to world amity, will have something more fundamental to contribute than blind speed and a nerve-wracking staccato. Perhaps one means lies in the transmission of wider sidebands, more overtones, typifying more of the nuances of living. Bell showed the way in transmitting not alone the words, but the warmth, of the human voice. Television, which breaks the strangle hold of time on the motion picture's projection of the human personality, will some day be an international accomplishment. Each step forward of this kind will bring the rulers of the peoples, their leaders, and eventually the peoples themselves, closer together.

The possibilities are so intriguing, may one be forgiven for romancing? Under conditions of idealized communication, diplomacy, if bungled at all, will be bungled at the top. For parliaments will no longer *ratify* treaties, they will *make* them, in person, through trans-oceanic, 24-hour, two-way, full-color television, in three dimensions on a 30-foot screen, with binaural sound channels, equipped with privacy devices and, if desired, with English-French and French-English speech inversion. The transmission, in all likelihood, will be at Government rates. Cigar smoke can be blown in locally at both ends of the circuit, completing all arrangements except the handshaking which will have to be delegated to the diplomatic corps. The documents themselves, including signatures will, of course, be exchanged by facsimile before the ink is dry. Because of its simplicity, the whole process can be repeated as often as the treaties so concluded are denounced by one of the parties!

V

I have digressed, and I wish to conclude in serious vein. We radio-and-electronic engineers inherit from the past an unbroken line of tradition of self-effacing service for the benefit of mankind. Where there have been shortcomings they have been human, understandable. Morse and the whole long list of his predecessors in physics and his successors in engineering did their best to add to the common man's knowledge and his powers. After this war you of my audience are going to continue in that tradition in the fields of science, communication, and industrial electronics. But in the meantime something has been added to the engineering profession; it has gathered new responsibilities with the years. More and more you will be called upon not merely to produce new implements for the exercise of power but, as an engineer, to influence their use and to prevent their abuse. Whether our postwar civilization is to be better or worse, whether there will ensue a hiatus of preparation for another struggle or, on the contrary, a long period of peace and contentment, will depend not so much upon the tools of progress which you indiscriminately place at the disposal of your fellow men, as upon the professional influence, individually and in concert with other engineers, which you exert upon their desire to utilize them for the greater good. To the extent that you meet with a measure of success, may the engineering profession, in taking credit, be as humble and as thankful as was Morse a hundred years ago today: "What Hath God Wrought!"

Design of Electronic Heaters for Induction Heating*

J. P. JORDAN†, NONMEMBER, I.R.E.

Summary—**The design of vacuum-tube oscillators for induction heating involves many factors not usually encountered in radio practice. The choice of the circuit to be used, the tank current, operating frequency, and the physical construction are dependent upon a knowledge of the theoretical and practical requirements of induction heating, and an understanding of the operating conditions to be met in the average factory. It is concluded that either the Colpitts or the coupled-grid circuit meets the requirements, and an operating frequency in the neighborhood of 500 kilocycles, with a tank current of from 100 to 300 amperes is desirable. In this paper, the induction-heating theory necessary to substantiate the above conclusions together with a brief discussion of the other factors involved is presented.**

Introduction

IT IS odd that the equipment first used extensively in the manufacture of electron tubes should now be using these electron tubes so widely to fill a need in the industrial heating field. The so-called "bombarders" have been used since the early days of tube manufacture to heat the internal elements during evacuation; only recently have electronic heaters been widely used as a source of heat in industrial processes. The cause was largely one of lack of suitable equipment; up to relatively recently such equipment was designed and built for a specific use in plants well acquainted with the operation of radio equipment, with little thought being given to the physical and operational requirements of the average industrial user. (See Fig. 1.)

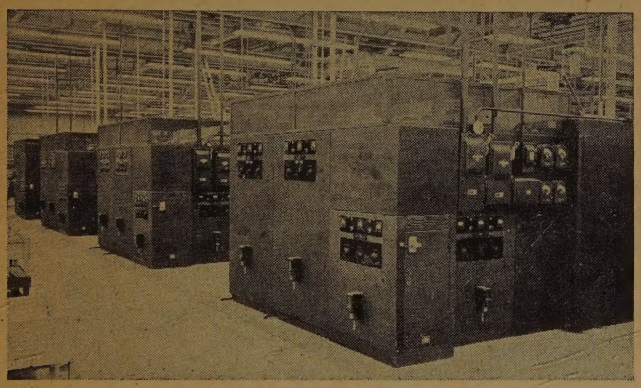

Fig. 1—Electronic-heater installation used in the manufacture of vacuum tubes at the General Electric vacuum-tube plant. Each group consists of two 5-kilowatt and two 15-kilowatt General Electric heaters.

The design and construction of suitable electronic heaters requires far more than a knowledge of oscillator circuits alone; it involves a search into the theory of eddy-current heating as applied to a wide variety of practical applications as well as an understanding of the requirements of the industrial user from the standpoint of cost, simplicity of operation, and physical sturdiness. Thus it is necessary first to examine the field in general and then to determine the operational characteristics of the electronic equipment required before attempting to judge the relative merits of the oscillator circuits available.

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† Industrial Heating Engineering Division, General Electric Company, Schenectady, New York.

Other Equipment in Use

High-frequency induction heating can be defined as the generation of sufficient heat in the surface of a conducting mass to produce a desired result.

Motor generators, operating at frequencies from 1,000 to 11,000 cycles, are widely used for melting metals and heating metals for forging and hardening. For such service, the power requirements are usually high and depths of penetration of the heat are not too critical. In fact, for melting, the surface-heating effect is undesirable; while for large shafts it is necessary that the energy input be fairly deep to avoid burning the surface before the required volume is heated.

Spark-gap equipment now in use is limited to frequencies of the order of 200,000 cycles and powers of approximately 15 kilowatts output. When properly applied and conservatively rated, they will give good service for hardening and brazing where the power requirements are not too heavy.

To date, no vacuum-tube oscillator has been built which can successfully compete economically with the motor-generator equipment in its frequency range. Thus, the field of application for such oscillators lies in those jobs where the higher frequencies are necessary; i.e., where the generation of heat around sharp contours or in shallow layers is required. (See Fig. 2.)

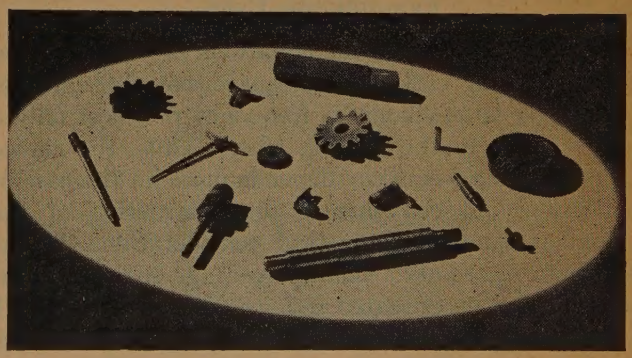

Fig. 2—Typical examples of brazed and hardened small parts.

Theory of Induction Heating

In induction heating, the heat is generated by I^2R losses produced by the eddy currents induced in the body. Hysteresis losses play a relatively minor part in the heating of magnetic materials. An approximate formula for such heating is

$$\Delta P = H_t^2\sqrt{\rho\mu f}/8\pi \qquad (1)$$

where

ΔP = power dissipated by eddy currents
H_t = tangential component of magnetic flux at surface of charge
ρ = resistivity of charge
μ = permeability
f = frequency.

Since eddy currents tend to flow on the surface rather than through a mass, a second factor must be considered, the depth of the peripheral layer in which the heat is generated. Thus p is the depth of the volume in which approximately 90 per cent of the heat is generated.

$$p = 5300\sqrt{\rho/\mu f} \tag{2}$$

where

p = depth of penetration of uniform magnetization centimeters
ρ = resistivity of charge ohm-centimeters
μ = permeability
f = frequency cycles per second.

Thus, from (1) and (2), we obtain

$$\Delta P = H_t^2\rho/16\pi^2 p. \tag{3}$$

Examining these equations, it will be seen that the rate of heat input is directly proportional to the square roots of the frequency and the resistivity, and inversely proportional to the depth of penetration factor p. Also note that the permeability which has a value of 1 for all nonmagnetic materials, can be of major importance when heating magnetic metals below the "curie temperature" which is that temperature above which magnetic properties cease to exist and μ becomes unity (1420 degrees Fahrenheit for low-carbon steel).

With the above background, it is now possible to outline those factors which must be considered when choosing an oscillator circuit for this application.

Frequencies Used

It will be noted from the equations previously given that the heating varies as the square root of frequency. A relatively large change in frequency produces only a minor change in the heating effect. Thus, the choice of an operating frequency depends more on the ease of application and convenience in construction of the equipment than on theoretical considerations. It has been found that only a small percentage of the applications thus far investigated require frequencies above 1 megacycle to obtain the desired heating effect. This is fortunate, since the design of the inductor coils becomes increasingly difficult as the frequency is raised over the 1-megacycle point. At these higher frequencies (2 to 20 megacycles), the heater coil and its leads become more and more a major element in the oscillator circuit, requiring operating personnel with a knowledge of radio circuits not generally found in the average industrial plant. Frequencies below 200 kilocycles are not used generally because of the size and cost of the necessary capacitors and chokes. The optimum operating frequency for the majority of applications is, therefore, in the neighborhood of 500 kilocycles.

Load Characteristics

The output requirements are to some extent unique. In order to generate eddy currents of the desired magnitude in many parts, it is necessary to use currents of the order of 100 to 300 amperes. In a few applications even higher currents are desirable; up to 1500 amperes have been used. These currents must flow through coils which vary over a wide range of inductance and resistance. Also, as the part heats up, the electrical characteristics at the heater-coil terminals change due to the increased resistance of the charge and the loss of magnetism when heating steel. Thus, the reflected resistance and inductance change rapidly with time during the heating cycle.

Factors Influencing Physical Features

The possible location of such equipment and the probable training of the men who will be expected to operate it must also be considered. In general, since this is an industrial heat-treating tool, it can be expected that it will be placed either in the heat-treating room in close proximity with furnaces, ovens, lead pots, cyanide pots, etc., or directly in the factory processing lines for such uses as brazing, soldering, forging, and hardening. Thus, it must be operable under the conditions of dust, dirt, fumes, and vibration usual to such locations. Likewise, since no radio technician will be available in most cases, it is necessary that the circuit be as simple as possible to permit routine maintenance by the average plant-maintenance crew and to provide the maximum of reliability.

The above considerations also affect the type of controls that must be provided. They must be simple. A factory operator should not be expected to be able to adjust a number of controls to obtain a required power output. The optimum condition, which can be approached but not always obtained, is to supply a single ON-OFF push button with perhaps a single control to produce the desired heating effect. (See Figs. 3, 4, and 5.)

Circuit Considerations

Thus, in the oscillator circuit to be used in this field the following conditions must be met:

1. Low cost—initial and maintenance;
2. Sturdy construction;
3. Simplicity of operation and maintenance;
4. Operating frequency should be of the order of 500 kilocycles;
5. Capable of operating into a load of variable resistance and inductance without the necessity for adjustment;
6. Output currents in the order of 100 to 300 amperes should be available.

A review of the above would indicate the self-excited oscillator, since its simplicity permits low cost and sturdy

construction while possible breakdowns are reduced by the elimination of extra elements. Because of the variation of the load characteristics during the heating cycle,

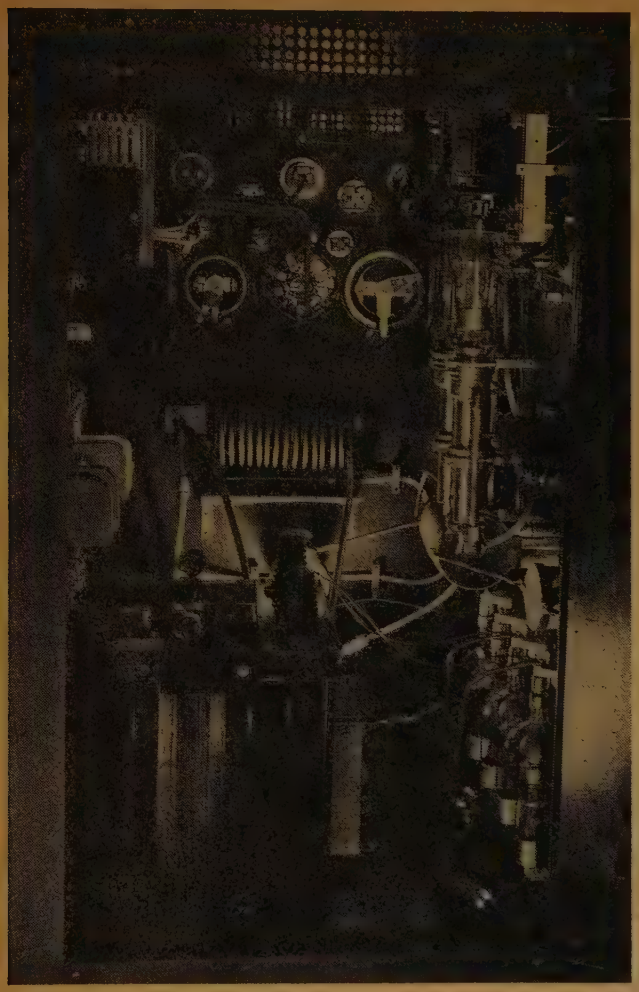

Fig. 3—Inside view of the construction of a 15-kilowatt electronic heater.

Fig. 4—Five-kilowatt electronic heater for high-frequency induction heating of metal parts. Worktable attached.

a fixed-frequency device is ruled out and the obvious place for the heater coil is directly in the tank circuit, thus permitting the oscillator to compensate for these changes by shifting frequency. This requirement also

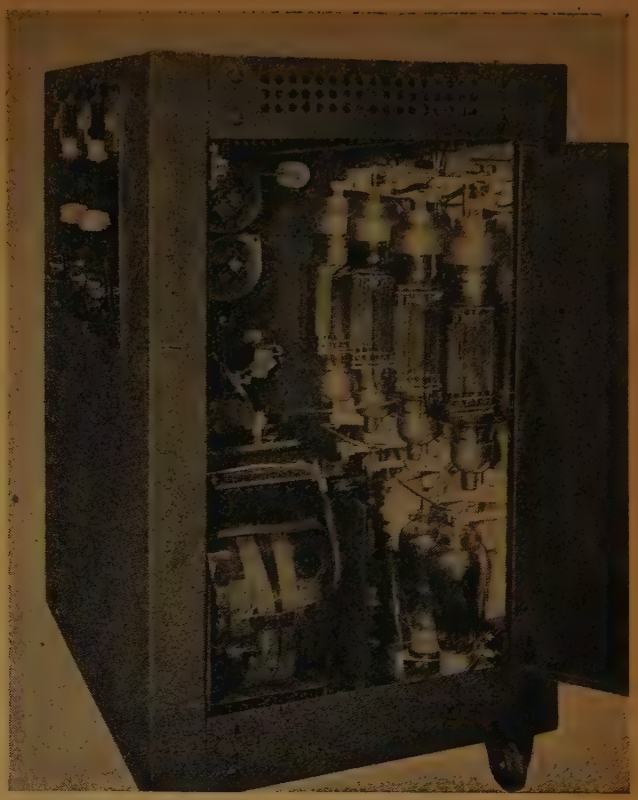

Fig. 5—Five-kilowatt electronic heater, showing inside construction.

eliminates the possibility of using a series of coupled tuned circuits to step up the current; in fact, it has been observed that by using three coupled resonant circuits between the oscillator and the load, it is impossible to heat a piece of steel above the magnetic point because of detuning as the load characteristics changed.

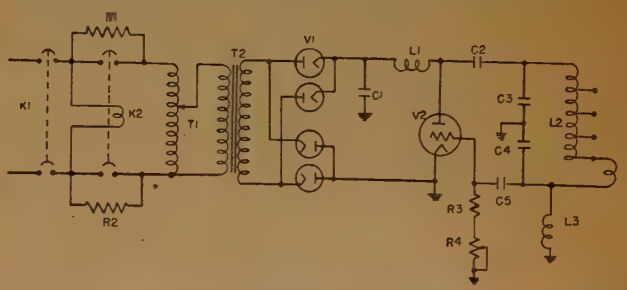

Fig. 6—Colpitts-oscillator circuit. (Radio symbols.)

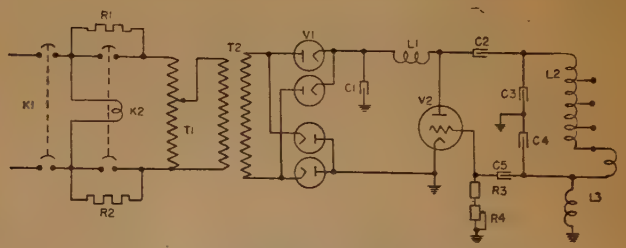

Fig. 7—Colpitts-oscillator circuit. (Electrical engineering symbols.)

The Colpitts-oscillator circuit (Figs. 6 and 7) which uses a split tank capacitor to obtain the necessary grid feedback voltage, and the inductively coupled grid oscillator circuit (Figs 8 and 9) fill most of the above

requirements. It is, of course, necessary for efficient operation that the voltage applied between grid and cathode be approximately 180 degrees out of phase with the plate-to-cathode voltage. Thus the Hartley oscillator, which utilizes a split tank inductance for the grid voltage is ruled out because the load must, of necessity, be coupled into part of the tank-circuit inductance which would affect the grid-phase relationship.

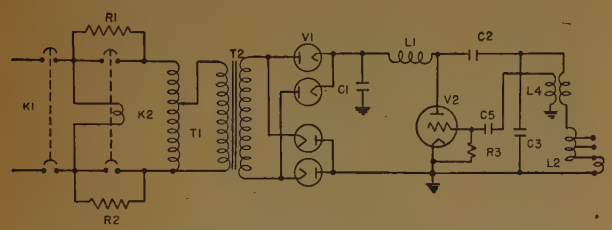

Fig. 8—Coupled-grid oscillator circuit. (Radio symbols.)

Of these two circuits, the Colpitts has two advantages; 1. Somewhat greater stability, since the grid phasing is always constant; 2. Fewer controls, since the grid excitation is always fixed. The coupled-grid circuit, however, has the advantage of the use of a lower cost capacitor because of the elimination of the tap required in the Colpitts circuit. Also, in some applications, it is advantageous to be able readily to change the grid drive. Both circuits are in wide use today.

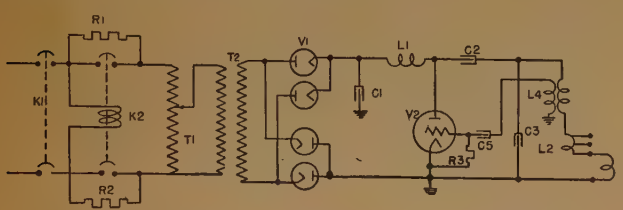

Fig. 9—Coupled-grid oscillator circuit. (Electrical engineering symbols.)

Figs. 6 to 9 show that coarse power control is obtained by use of an autotransformer (or tapped plate transformer) and fine control by varying the grid-bias voltage. Resistors 1 and 2 limit the voltage surge which occurs on closing the main contactor. They are short-circuited within a few cycles by contactor *K2*. Capacitor *C2* prevents any direct voltage from appearing on inductor coils *L2*, while inductance *L1* and capacitor *C1* block the radio-frequency voltage from the rectifier tubes. Inductance *L3* is a safety bleeder choke to provide a direct-current ground for the output inductance.

It will be noted that in the Colpitts circuit, inductance *L2*, which includes the heater coil and an additional internal inductance, is not directly grounded at one end as in the coupled-grid circuit but has a radio-frequency ground point due to the ground between capacitors *C3* and *C4*. Approximately 1/5 of the total radio-frequency tank voltage appears between this point and the cathode end of the inductance. When using small heater coils this is a disadvantage, since both sides of the coil are at a higher voltage to ground than in the inductively coupled grid circuit. However, for larger coils, or for series connection of a number of small ones, it is very advantageous since it greatly reduces the maximum voltage to ground appearing on them. Incidentally, the trend today is to use liquid-filled tank capacitors rather than mica, since the better stability and loss characteristics of mica are unimportant compared to their inability to withstand overloads.

Tubes Used

Both air- and water-cooled tubes are in use today, each having their disadvantages. The air-cooled tubes are considerably higher in cost and require considerable maintenance in the cleaning of the air filters. With water-cooled tubes, some difficulty is encountered with condensation during humid summer days. Little difficulty has been encountered due to scale formation in the use of the average city water supply for cooling such tubes.

The use of rectifiers rather than self-rectifying circuits is almost universal since the high cost of the average oscillator tube makes it necessary to use them as efficiently as possible. A rectifier bank more than saves its cost in increasing the output kilowatt hours available from the oscillator tubes.

The tube cost is a very important item in these equipments and the average industrialist examines it in terms of cost per hour of operation. With an expected life of 2000 hours at full rating, the replacement cost of some tube complements averages more than $2\frac{1}{2}$ cents per kilowatt-hour. By derating these same tubes and reducing the filament voltages, life can be extended to 10,000 hours and the cost drops to 0.6 cent per kilowatt-hour. However, it is envisioned that when conditions permit, a new line of tubes will be developed for this use with lower initial cost and longer life ratings.

Symbols

C1 = bleeder capacitor
C2 = direct-current blocking capacitor
C3 = tank capacitor—plate section
C4 = tank capacitor—grid section
C5 = grid direct-current blocking capacitor
K1 = main power contactor
K2 = step start contactor
L1 = high-frequency blocking choke
L2 = tank inductance
L3 = safety bleeder choke
R1, *R2* = surge-limiting resistors
R3 = grid-biasing resistor
R4 = grid-biasing resistor (variable)
T1 = autotransformer
T2 = plate transformer
V1 = rectifier tube bank
V2 = oscillator tube

Need for an Instrument to Measure pH in Localized Areas of the Mouth*

B. O. A. THOMAS†, NONMEMBER, I.R.E.

Summary—**The loss of tooth structure in the cervical (neck) region of the tooth may occur as a result of (1) dental caries (decay), (2) abrasion (mechanical), (3) erosion (chemical), or (4) combinations of (1), (2), and (3). This may lead to the condition known as "sensitive neck" of the tooth. Lost structure in the cervical area, just as in other parts of the tooth, must be replaced by means of fillings. Our problem is to prevent cervical tooth destruction and its subsequent unpleasant sequelae.**

The shallow, narrow gingival crevice around the neck of the tooth contains a fluid, the pH of which may or may not agree with that of the saliva. An acid fluid may decalcify the tooth structure with which it is in contact in the cervical region. Since the great majority of lesions occur on the side of the tooth next to the lip or cheek, does the fluid in the gingival crevice vary in pH locally in relation to a single tooth?

The problem demands a much more sensitive indicator than the litmus papers which are in present use. The instrument must be capable of use directly in the oral cavity, and must require a minimum of fluid.

The author has reviewed the literature concerning technics for measuring pH, and has discussed the subject with a number of investigators who are using such methods. Since these methods do not give promise of easy adaptability to the present problem, they are not discussed here.

DENTISTRY is a young profession. We are justly proud of what has been accomplished by clinical and laboratory research during this first one hundred years of organized dentistry. A very great deal of this progress has been possible as a result of borrowing methods and instruments from other fields of scientific research. Examples of this are the use of radioactive isotopes, used in studying the interchange of mineral salts in calcified tooth structure; various applications of the Roentgen ray and radium; and the use of vital stains in studying the growth and development of bone and teeth.

However, we realize that the surface has hardly been scratched and that the two fundamental dental problems, dental caries (decay) and periodontoclasia (pyorrhea) are far from solution.

Dental caries are well known to all. There is marked individual variation in susceptibility to them. The disease may attack any or all surfaces of teeth. The proximal and occlusal (masticating) surfaces are most often attacked, and in these regions, the sequence of development of the lesions is remarkably constant. However, in the cervical region, or neck, of the tooth, the loss of tooth substance may develop in any of several ways: (1) as caries proper; (2) as abrasion (especially from toothbrush). The liquid dentifrice companies capitalize on this! (3) as chemical erosions, due to decalcification of the tooth structure in the cervical area. Once abrasion or erosion has destroyed some tooth structure, caries may sometimes be superimposed on the already existing lesion.

When some of the tooth structure of the cervix has been lost, the condition known by the layman as "sensitive neck" may develop. There are chemical and restorative methods of treating this condition, but naturally, its prevention is the final goal for which we are striving.

Fig. 1—Low-power photomicrograph showing a section of an upper molar tooth and its surrounding tissues.
E = enamel
D = dentin
P = dental pulp ("nerve")
P.M. = peridental membrane
B = bone
G = gingiva (gum tissue)
G.C. = gingival crevice (area containing fluid which is discussed in the text)
C = crack (artifact produced during preparation of specimen).

There is a shallow crevice between the tooth and the gingival (or gum) tissue (Fig. 1). This crevice is filled with a fluid, the source of which has not been satisfactorily explained. The pH of this fluid, as tested with litmus paper, may or may not agree with that of the saliva (Figs. 2 and 3). It is felt by some investigators that if the fluid is of low pH value, the mere fact of its constant contact with the cervical tooth structure is sufficient to explain decalcification of that area. If this is true, this fluid must vary in its acidity or alkalinity in localized areas of the same tooth, because the very great majority of cervical lesions is seen on the buccal

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or labial surfaces, that is, the side of the tooth next to the cheek or lip.

Attempts to measure the pH of plaques of foreign material on teeth surfaces have been reported. Colorimetric methods were used. However, these methods are not satisfactory for working with fluid from the cervical

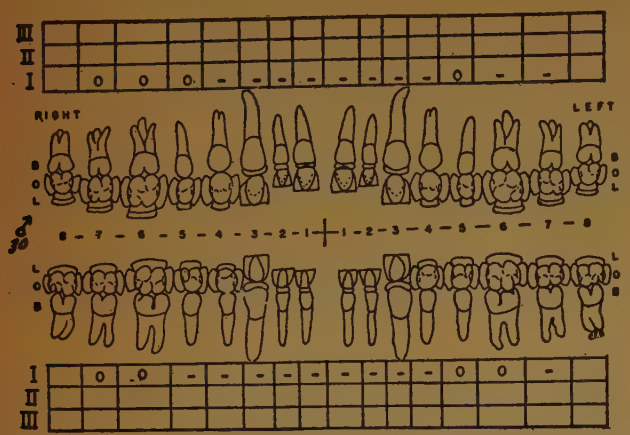


Fig. 2—Chart on which is recorded the results obtained by testing the pH of the fluid in the gingival crevice with red and blue litmus paper. Male, age 30 years. There has been no destruction of the teeth at the cervical ("neck") region in this case. (There is one amalgam filling, $\overline{7}|$, see chart.) This is considered an average, or normal case. + =acid; 0 =neutral; − =alkaline.

crevice because there is so little fluid present. At the present time we are working on the problem, using litmus papers, gathering data which may be of help later in evaluating the various conditions we see. We realize, however, that the litmus papers will merely give us an idea as to whether the fluid in the gingival crevice of a single tooth is on the alkaline or acid side of neutrality.

We believe an instrument, which could be used in the mouth to measure *directly* the pH of very localized areas, and requiring a minimum of fluid, would be of very great value in giving us some basic information in the problem of cervical tooth destruction.

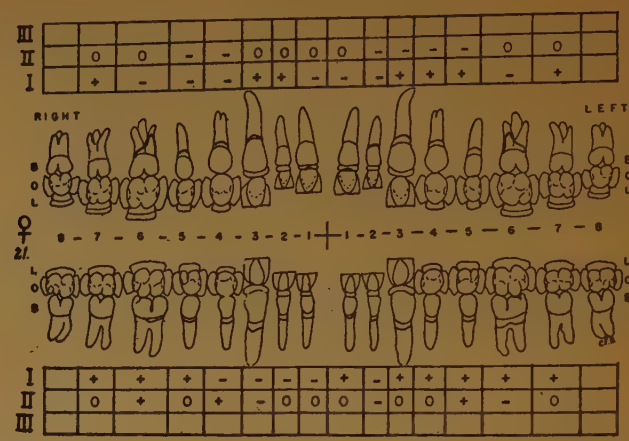


Fig. 3.—Same as Fig. 2. Female, age 21 years. This includes the record of two different sets of readings, taken on different days. The dark lines on the roots of some teeth indicate the degree of "recession of the gum tissue."

Such an instrument would be extremely useful for many other purposes, such as determining the pH of (1) secretions in the ducts of glands, (2) vaginal mucosa during various phases of the menstrual and pregnancy cycles, and (3) bone lesions during infection and healing. The investigators in widely divergent fields would be thankful to have such an apparatus available.

The Development of a New Station Location or Z-Marker Antenna System*

J. C. HROMADA†, ASSOCIATE, I.R.E.

Summary—This paper presents descriptive and flight-test information relating to a new type Z-marker antenna system which possesses a number of marked advantages and improvements over that used at present Z markers. The antenna consists of two spaced-dipole arrays, crossed at right angles to each other, and excited in quadrature time phase.

The design of the arrays is based on the principle of proportioning dipole currents in accordance with the coefficients of the successive terms of the binomial expansion.

The antenna is of simple and rugged design, capable of maintaining a high degree of marker-zone stability under rain, snow, and sleet conditions, and lending itself to prefabrication in units and sections of transmission line for ease in field installation.

The marker zone is considerably narrower than the zone provided by present Z markers and is ideally suited to instrument approaches. Whereas the height of present Z markers is limited to about 10,000 feet, the new marker antenna may readily be extended to over 20,000 feet altitude with present transmitting equipment.

Pilots have, heretofore, noted the apparent excessive broadening of the marker zone during flights off-course or on flights over the radio range at a large crab angle. This effect is greatly reduced with the new antenna, making it possible for the pilot to obtain a more accurate fix on the range station.

Introduction

STATION location or Z markers used at radio range stations serve as a positive indication of the cone-of-silence of the range and are the result of development work carried on by engineers of the Civil Aeronautics Administration during the years 1934 and 1937. The first of the present commercially manufactured Z markers was installed on the Federal Airways and commissioned for regular service in 1938.[1] These 5-watt markers emit a cone of radio-frequency energy directed

* Decimal classification: R526.1. Original manuscript received by the Institute, February 2, 1944.

† Civil Aeronautics Administration, Washington, D. C.

[1] W. E. Jackson and H. I. Metz, "The development, adjustment and application of the Z marker," Civil Aeronautics Authority, Technical Development Report No. 14, July, 1938.

vertically. The signal is transmitted continuously at 75 megacycles and is modulated with a 3000-cycle tone. The experience gained during the past several years has proved these markers to be a valuable complement to the Federal aids to air navigation.

The demand for increased stability, greater accuracy as a point of fix from which to begin the letdown procedure, and an increased height of signal zone to insure reception at higher altitudes, has indicated the need for an improved antenna system.

It is the purpose of this paper to describe an experimental Z-marker antenna system based on the radiation properties of spaced dipoles, having desirable features which will materially aid in increasing the accuracy of aircraft navigation, particularly during letdown procedures.

Design and Description

Greater stability of the Z-marker zone can be accomplished by an antenna structure which is more rigid mechanically than the one used at present. Separating the individual elements of the antenna and their feeders to decrease their mutual coupling will facilitate the adjustment of the array and insure the stability of the final adjustment of the system. An increase in altitude of the zone can be accomplished by increasing the power in the present antenna, but in so doing the radiated pattern will be expanded in all directions, resulting in a substantial broadening of the radiated beam at low altitudes. The accomplishment of an increased altitude of zone with a narrower zone at low altitudes necessitates redesigning the antenna system in order to provide greater vertical gain and sharper vertical directivity.

Increasing the gain and directivity of antenna arrays by means of spaced antennas, reflectors (both driven and parasitically excited), and stacked arrays is well known to the art. The utilization of these principles for the improvement of Z-marker antennas has been suggested by Green.[2] Sharpness of vertical radiation may be obtained by using several rows of parallel dipoles spaced one-half wavelength apart in the rows and spacing the rows one wavelength apart. Green shows that in order to obtain substantial symmetry between the radiation in the vertical plane containing the radiators and in the vertical plane at right angles to the radiators, the antenna array should form a square figure in the horizontal plane. The numerical progression of rows of radiators and radiators per row is as follows: 1-2, 2-4, 4-8, etc., respectively, where the first figure in the group indicates the number of rows and the second figure the number of radiators in each row. The total number of radiators increases as 2^{2n-1} where n, an integer, may be considered the number designating the successive stages in the expansion of the system. Thus, the third stage would require a total of 32 radiators, the 4-8 combination. Aside from the difficulties of construction and adjustment of such a large number of radiators, the second stage of expansion, the 2-4 array, provides a vertical field pattern having two minor lobes which are 27 per cent as large as the major lobe, a condition which nullifies its usefulness as a zone marker. The 4-8 combination has six minor lobes, the greatest of which is approximately 23 per cent of the major lobe. A means of reducing the minor lobes offered by Green is to stack the antennas in the vertical plane. Two layers of the 2-4 combination will double the number of radiators and reduce the secondary lobes to only 17 per cent. Three layers will triple the number of radiators and reduce the lobes to only 10 per cent. It can easily be seen that the system proposed is inherently complex mechanically and electrically.

John Stone Stone has shown[3] that a high degree of directivity may be secured from a rectangular antenna array by proportioning the currents along the rows and columns of the array in accordance with the coefficients of the successive terms of the binomial expansion

$$(a+b)^n = a^n + \frac{n}{1}a^{n-1}b + \frac{n(n-1)}{1\times 2}a^{n-2}b^2 \cdots \text{etc.}, \quad (1)$$

where $(n+1)$ = number of dipole elements in a row or column.

Fig. 1—Pascal's triangle diagram illustrating the development of cophased collinear arrays having high directivity based on relative dipole current proportioning in accordance with the coefficients of the successive terms of the binomial expansion $(a+b)^n$, where $n+1$ = number of dipoles.

(a) Single dipole element carrying unity current.
(b) Two dipoles, each with unity current, spaced 2S degrees.
(c) Three-dipole array, relative currents 1-2-1, evolved from two (b) arrays spaced $2S$ degrees.
(d) Four-dipole array, relative currents 1-3-3-1, evolved from two (c) arrays spaced $2S$ degrees.
(e) Five-dipole array, relative currents 1-4-6-4-1, evolved from two (d) arrays spaced $2S$ degrees.

This process can be carried further to more extensive arrays. For spacings $2S = 180$ degrees, only the major lobe appears in the directive patterns. For some applications the spacing may be increased considerably beyond 180 degrees, resulting in greater sharpness of the major lobe, provided that minor lobes of a few per cent can be tolerated.

For example, let us assume an array of three cophased collinear dipoles. Then, $n=2$, and the relative amplitudes of the currents in the three dipoles would be $1+2/1+2(2-1)/(1\times 2)$ or $1-2-1$. For four collinear dipoles, the relative amplitudes would be $1-3-3-1$ and for five dipoles the relative amplitudes would be $1-4-6-4-1$. The relationship between dipole currents and the number of dipoles may be represented by Pascal's triangle, as shown in Fig. 1. This figure also illustrates the development of the binomial array.

[2] A. L. Green, "Marker beacons for symmetrical radiation," *Amalgamated Wireless (Australia) Tech. Rev.*, vol. 3, pp. 113–142; January, 1938.

[3] United States Patent No. 1,643,323.

The free-space radiation patterns in the vertical plane for several arrays of half-wave dipoles are shown in Fig. 2 and illustrate the application to spaced arrays of the principle of currents proportioned in accordance with the coefficients of the successive terms of the binomial expansion. Fig. 2 also shows the vertical patterns obtained with two-, three-, and four-element arrays with unity current ratios. Considering only dipole spacings of 180 degrees, the two-element array with equal currents has a substantially wide pattern with no minor lobes. The addition of a third dipole in line, spaced 180 degrees from an adjacent dipole and having a current amplitude equal to the other two dipoles, provides a sharper pattern but results in the addition of two minor lobes which are opposite in phase to the major lobe and have a maximum intensity of 11.7 per cent of the major lobe radiation. When the three dipole currents are proportioned in a 1–2–1 ratio, the minor lobes are entirely suppressed and the main lobe becomes slightly broader. The major-lobe sharpness may be increased by the use of four dipoles spaced 180 degrees and carrying equal currents. At an angle of 20 degrees from the vertical, the strength of the radiated signal may be reduced to 55.4 per cent of the value obtained with the 3-element 1–2–1 array. Minor lobes again develop which have a maximum value of 17 per cent of the maximum of the major lobe. These minor lobes may be eliminated, as before, by the proportioning of the dipole currents in a 1–3–3–1 ratio. Although the minor lobes are eliminated the main lobe becomes somewhat broader, since the radiation at an angle of 20 degrees from the vertical has been increased by 55.5 per cent over that obtained with unity current in the four dipoles. However, the signal strength of the 1–3–3–1 array at an angle of 20 degrees from the vertical is 86 per cent of the signal obtained from the 1–2–1 array.

The radiation characteristic in the vertical plane which includes the elements of a four-element array with a 1–3–3–1 distribution of currents is practically the same as that of a three-element array with a 1–1–1 distribution of currents, except that the minor lobes have been suppressed. The sharpening of the pattern can be carried on further by the addition of more dipoles and by the proper proportioning of the currents. The selection of the desired array becomes a problem of economy and compromise. In general, the intensity of the minor lobes increases with increase in the number of radiators unless the currents are properly proportioned. In actual service, where the radiators are located a quarter wavelength above a metallic counterpoise, the free-space radiation characteristic will be modified by the factor

$$\sin (90 \sin \theta). \qquad (2)$$

It is possible to design an array in which the minor lobes are only partially suppressed below a tolerable value. However, the limit of expansion of the array is governed by cost and space limitations, which increase with an increase in the number of dipoles. In this development it was decided to limit the array to four dipoles.

A study was then made to determine the limits to which the radiation pattern could be sharpened by spacing the elements of the array at distances greater than 180 degrees without impairing the usefulness of the system as a zone marker. Fig. 2 also illustrates the patterns obtained from the 1–2–1 array with dipole spacings of 180, 200, 220, and 240 degrees. It is seen that as the spacing increases, the pattern becomes sharper with a slight tendency toward the production of minor lobes. It is interesting to observe that the 1–2–1 array, using 220-degree dipole spacing, has a sharpness equal to the 1–3–3–1 array, using a spacing of 180 degrees. The maximum value of the minor lobe obtained with the 1–2–1 array and 220-degree spacing is only 1.6 per cent of the maximum value of the major lobe for free-space radiation. This is reduced to 0.64 per cent when the array is located a quarter wave above a metallic counterpoise. A spacing of 240 degrees shows a slightly sharper main lobe pattern with 4.1 per cent minor lobes for free-space radiation. The minor lobe amplitude reduces to 2.2 per cent when the array is located a quarter wave above a metallic counterpoise. The minor lobes are present with spacings of 200 and 220 degrees, but they are too small to be indicated on the polar co-ordinate paper.

Subsequent radiation patterns differ from those shown in Fig. 2 in that the reflections from the earth's surface, assuming perfect conducting ground, are taken into consideration. Also, in Figs. 2, 3, and 4, the radiation patterns have all been plotted to a maximum of unity at 90 degrees from the horizontal plane. The patterns, therefore, do not show the gain of one array over the other and are not comparable on the basis of constant power input to the arrays. The numerical figures shown above the radiators on the figures are relative amplitudes of the current values in the elements of each array.

Fig. 3 shows the pattern for the Z-marker array in present use at radio range stations. The array consists of two collinear, cophased dipoles carrying equal currents and spaced 180 degrees. In addition to the radiation pattern, $f_A(\theta)$ in a vertical plane containing the radiators, the radiation pattern $f_B(\theta)$, in a vertical plane normal to the line of the radiators, also is shown. These planes henceforth will be referred to as the A and B planes. The solid of radiation would resemble a thick cactus leaf and the distance from the origin to any point on its surface would represent the relative field strength at that particular angle measured at a constant distance from the origin. The direction of the electric potential vector at any point is at all times in a plane containing the radiators and the point in question and is normal to the line joining the point with the origin. Thus, in the A plane the direction of the electric-potential vector varies from vertical at the earth's surface to horizontal above the array. In the B plane the electric vector is at all times horizontal and parallel to the radiators. Since airplanes usually fly through the marker zones at a constant altitude with the receiving antenna in a horizontal

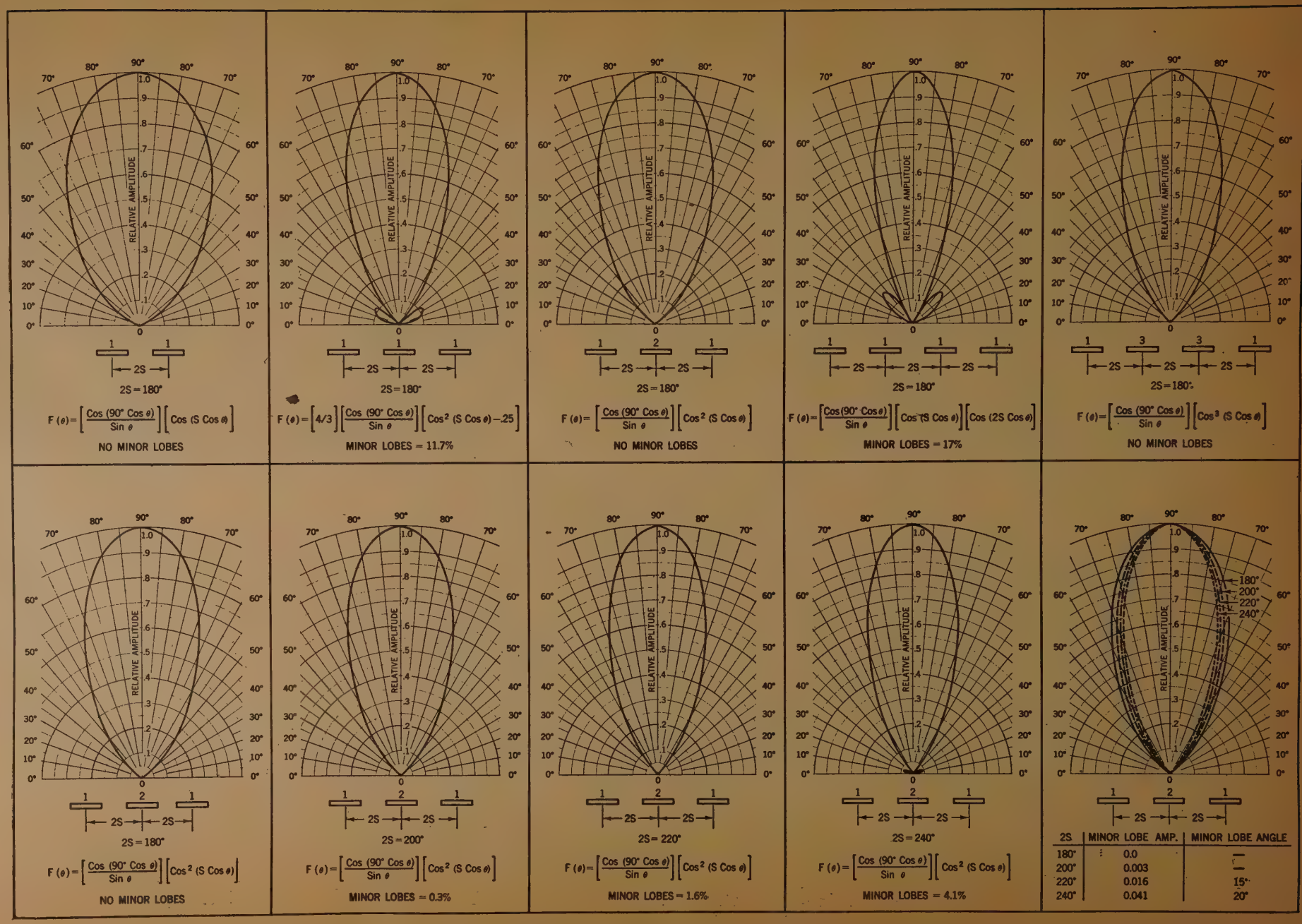

RELATIVE AMPLITUDE
1 1
2S
2S = 180°
F(θ) = [Cos (90° Cos θ) / Sin θ] [Cos (S Cos θ)]
NO MINOR LOBES
1 1 1
2S 2S
2S = 180°
F(θ) = [4/3] [Cos (90° Cos θ) / Sin θ] [Cos² (S Cos θ) − .25]
MINOR LOBES = 11.7%
1 2 1
2S 2S
2S = 180°
F(θ) = [Cos (90° Cos θ) / Sin θ] [Cos² (S Cos θ)]
NO MINOR LOBES
1 1 1 1
2S 2S 2S
2S = 180°
F(θ) = [Cos (90° Cos θ) / Sin θ] [Cos (S Cos θ)] [Cos (2S Cos θ)]
MINOR LOBES = 17%
1 3 3 1
2S 2S 2S
2S = 180°
F(θ) = [Cos (90° Cos θ) / Sin θ] [Cos³ (S Cos θ)]
NO MINOR LOBES
1 2 1
2S 2S
2S = 180°
F(θ) = [Cos (90° Cos θ) / Sin θ] [Cos² (S Cos θ)]
NO MINOR LOBES
2S = 200°
F(θ) = [Cos (90° Cos θ) / Sin θ] [Cos² (S Cos θ)]
MINOR LOBES = 0.3%
2S = 220°
F(θ) = [Cos (90° Cos θ) / Sin θ] [Cos² (S Cos θ)]
MINOR LOBES = 1.6%
2S = 240°
F(θ) = [Cos (90° Cos θ) / Sin θ] [Cos² (S Cos θ)]
MINOR LOBES = 4.1%
180°
200°
220°
240°
2S | MINOR LOBE AMP. | MINOR LOBE ANGLE
180° | 0.0 | —
200° | 0.003 | —
220° | 0.016 | 15°
240° | 0.041 | 20°

Fig. 2—Free-space vertical-radiation patterns for various numbers of cophased collinear dipoles, current ratios and spacings. Lower half of each pattern not shown.

position (below and in line with the axis of the fuselage), the patterns shown are not strictly representative of the voltage impressed on the receiving antenna as it is flown over the array. However, at high altitudes and near a point directly over the array, the effect of varying distance can be neglected. When an airplane is flying in

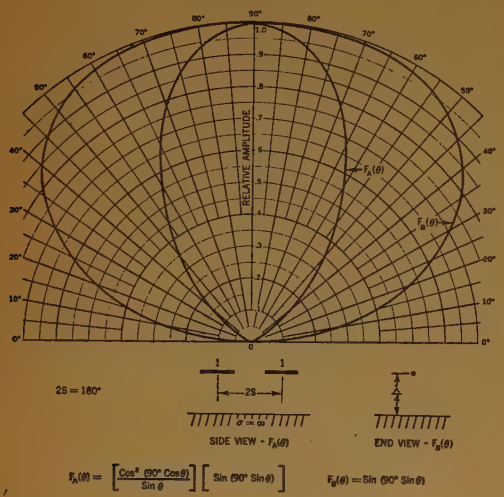


Fig. 3—Vertical radiation patterns; present Z-marker array.

the A plane, the $f_A(\theta)$ pattern is to be considered, with proper emphasis being given to the angle between the electric-potential vector and the receiving antenna. The voltage impressed on the receiving antenna is the product of the field strength and the cosine of this angle. The effect of this factor, which is equal to $\sin\theta$, where θ is the elevation angle measured from the horizontal, is to sharpen the $f_A(\theta)$ pattern at low angles with only a

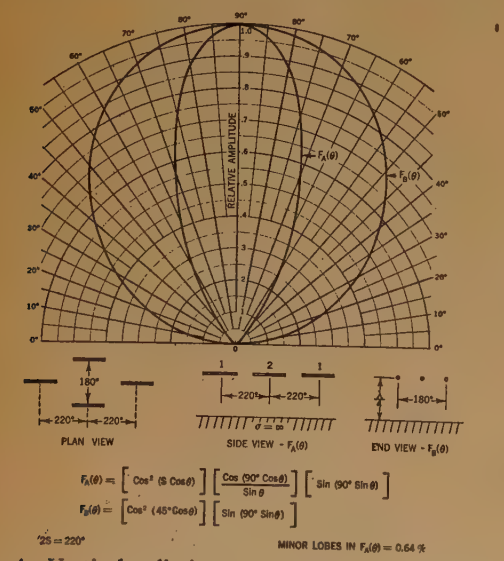


Fig. 4—Vertical-radiation patterns; spaced-dipole array.

slight effect at the high angles. When the airplane is flown parallel to the A plane and to one side of the array, the solid radiation pattern must be considered. However, at the instant it enters the B plane, the $f_B(\theta)$ pattern without modification represents the voltage impressed on the receiving antenna, since the electric-potential vector is parallel to the receiving antenna. When these factors are borne in mind, Fig. 3 indicates that an airplane flying parallel to the array and to one side of a point directly over the antenna will enter a signal zone of sufficient intensity to give a "light-on" indication[4] at a greater distance to one side of the station than would be the case had the airplane flown through a point directly over the station. This phenomenon of the elongation of the "light-on" zone for flights parallel to the array and to one side results in a broad indication of the location of the Z marker. To reduce this broadening of the "light-on" zone, it is necessary to make the $f_B(\theta)$ pattern similar to the $f_A(\theta)$ pattern, neglecting, of course, the pattern of the receiving antenna. Although the $f_A(\theta)$ pattern of the 1-2-1 array (with 220-degree spacing) was considered satisfactory, its $f_B(\theta)$ pattern is identical with the $f_B(\theta)$ pattern of Fig. 3. To reduce the $f_B(\theta)$ pattern, the array was modified in such a manner as to obtain the electrical equivalent of the 1–2–1 array and thus maintain the $f_A(\theta)$ pattern. This was ac-

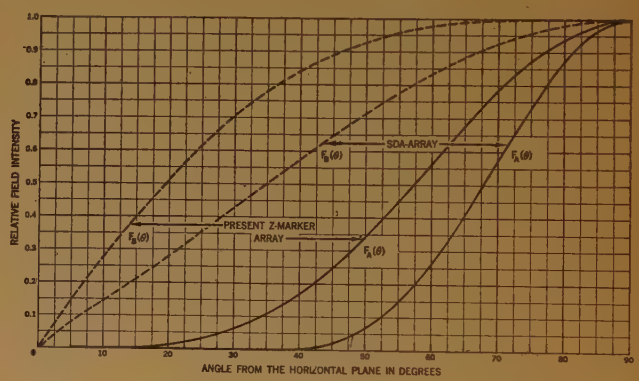


Fig. 5—Vertical-radiation patterns; antennas a quarter wave above perfect earth.

complished by splitting up the center element into two parallel dipoles, spaced 180 degrees and carrying equal currents. The net result is a four-dipole array, a plan view of which is shown in Fig. 4. The $f_A(\theta)$ and $f_B(\theta)$ patterns are illustrated in the same figure. The minor lobes of the $f_A(\theta)$ pattern are too small (0.64 per cent) to be drawn on polar co-ordinate paper. Fig. 5 shows the patterns of Figs. 3 and 4 plotted in rectangular co-ordinates and gives a clear picture of the degree of sharpness effected by the new spaced-dipole array (*SDA*). It is also seen from Figs. 3, 4, and 5 that the sharpening effect, through the use of the spaced-dipole array design, has been greater in the $f_A(\theta)$ patterns than in the $f_B(\theta)$ patterns. Fig. 5 also indicates that the present Z marker is more symmetrical than the spaced-dipole array. In practical terms this signifies, as will be shown later in discussion of the results, that the reduction in the elongated pattern, though substantial, was not so great as the reduction in the zone width.

[4] The term "zone" is used to designate that region above the Z-marker antenna in which can be found a signal of sufficient field strength to light the Z-marker instrument light in an airplane. This region is often referred to as the "light-on" zone and is the useful region of the radiated beam.

During the early stages of development of this array, the end dipoles were also displaced so that each one was collinear with one of the center dipoles. This arrangement resulted in a greater narrowing of the $f_B(\theta)$ pattern than that obtained with the final arrangement and in a greater reduction in the elongated pattern when flying parallel to the array and to one side. However, because of the arrangement of dipoles, the array was not symmetrical for angles measured either side of the normal to the collinear axis of the array in the horizontal plane. This dissymmetry resulted in a minor lobe of 17.5 per cent for radial flights over the array in a plane which made a 120-degree angle with the axis of the array. The maximum of the lobe occurred at an elevation angle of

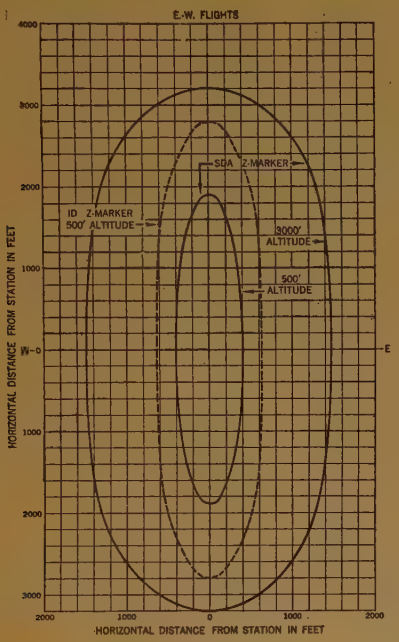


Fig. 6—Parallel-chord patterns of Z markers for 10,000-foot signal

27.5 degrees. At 40 degrees it is still 12 per cent. This minor lobe was quite apparent both in the headset and on the marker-indicator lamp for parallel-chord flights to either side of the array at an altitude of 500 feet. For this reason the unsymmetrical arrangement was discarded. The final arrangement of the spaced-dipole array has a minor lobe of 5.65 per cent for a 45-degree radial flight through the marker. However, this lobe lies along a 21-degree elevation angle. The horizontal distance for an altitude of 500 feet at 21 degrees elevation angle is 1374 feet. Examination of the elongated patterns in Fig. 6 will show that this distance out along a 45-degree radial is well outside the "light-on" zone of the marker. As the airplane is moved in nearer the array, the elevation angle increases. This fact may explain why no minor lobes are detected on parallel chord flights at an altitude of 500 feet either side of the symmetrical spaced-dipole array. The general equation for this array is

$$F(\theta)_\alpha = 1/2\,[\cos(A\cos\theta) + \cos(B\cos\theta)]$$
$$[\cos(90^\circ\cos\alpha\cos\theta)/\sqrt{1-\cos^2\alpha\cos^2\theta}\,][\sin(90^\circ\sin\theta)] \quad (3)$$

where $A = 220$ degrees $\cos\alpha$

$B = 90$ degrees $\sin\alpha$

θ = the elevation angle measured from the horizontal plane

α = the angle the vertical plane under consideration makes with the collinear axis of the array.

The first term of $F(\theta)_\alpha$ is the array factor, the second the form factor of the dipoles, and the third the reflection factor of the ground or counterpoise.

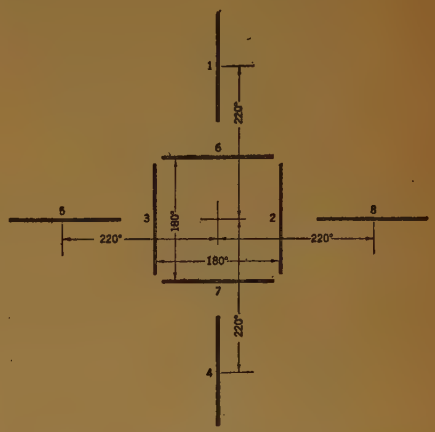


Fig. 7—Plan layout of the spaced-dipole Z-marker antenna.

A plan diagram of the spaced-dipole antenna is shown in Fig. 7. The antenna consists of two of the arrays shown in Fig. 4, crossed at right angles and fed in quadrature time phase with respect to each other. The central equivalent element of each 1-2-1 array is composed of two dipoles spaced one-half wavelength apart laterally, with each carrying unity current. Dipoles 1, 2, 3, and 4 constitute one array, while dipoles 5, 6, 7, and 8 constitute the other array. The dipoles are individually supported on pedestals a quarter wavelength above a 30-foot-square counterpoise consisting of 3-inch-square galvanized iron-wire mesh. The counterpoise is supported $6\frac{1}{2}$ feet above the ground, and is similar to that used on present Z markers.

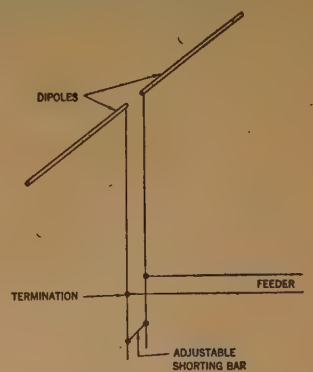


Fig. 8—Dipole antenna and tuning section.

The radiating elements of the antenna are horizontal dipoles of $\frac{1}{8}$-inch copper tubing and are fed at their centers by means of 140-ohm dual-shielded transmission line. A schematic diagram of a dipole is shown in Fig. 8. Since the dipoles are fed at a point of low potential, they

are inherently free from insulation difficulties and make for a stable system which is little affected by rain, snow, or ice. A spreadout diagram of the feeding system is shown in Fig. 9. Stub sections of transmission line *TA*

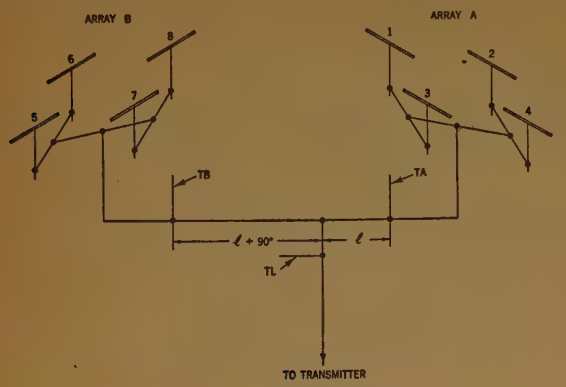


Fig. 9—Schematic diagram of spaced-dipole Z-marker antenna system.

and *TB* are used to terminate the common feeders to the two arrays. The common feeder to array *B* is 90 electrical degrees longer than the feeder to array *A* to provide the necessary quadrature phase relationship. A 2:1 stub section *TL* terminates the main feeder line. A view of the experimental antenna system is shown in Fig. 10.[5]

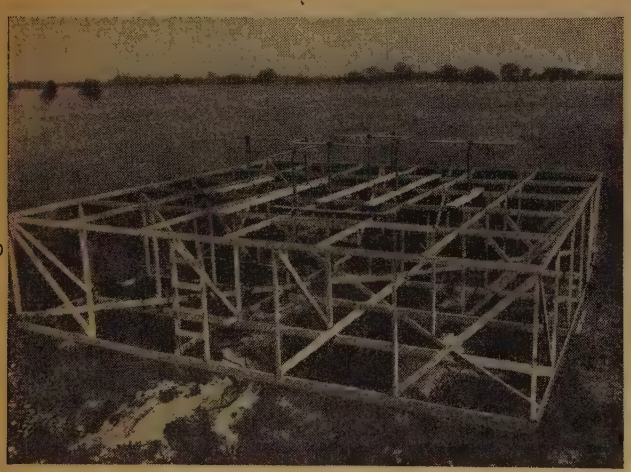


Fig. 10—View of the experimental spaced-dipole Z-marker antenna system.

Flight Tests

The power into the spaced-dipole Z-marker antenna was adjusted to produce the same height of signal zone as that obtained from the present Indianapolis (*ID*) Z marker, and the gain of the receiver was adjusted so that the height of the signal zones of both markers was limited to approximately 10,000 feet above ground. Comparison flights were then made on the two markers and the width of the "light-on" zone was measured. Direct measurements of the light time with a stop watch were not considered accurate since the apparent brilliancy of the light is affected by the normal lighting in the airplane cabin, thus making it difficult to judge the exact brilliancy at which to begin and end the timing. It was therefore found expedient to measure the light time by means of a recorder. The 3000-cycle tone output of the receiver was rectified and applied to the recorder. The time during which the rectified signal voltage was equal to or greater than a predetermined value was scaled

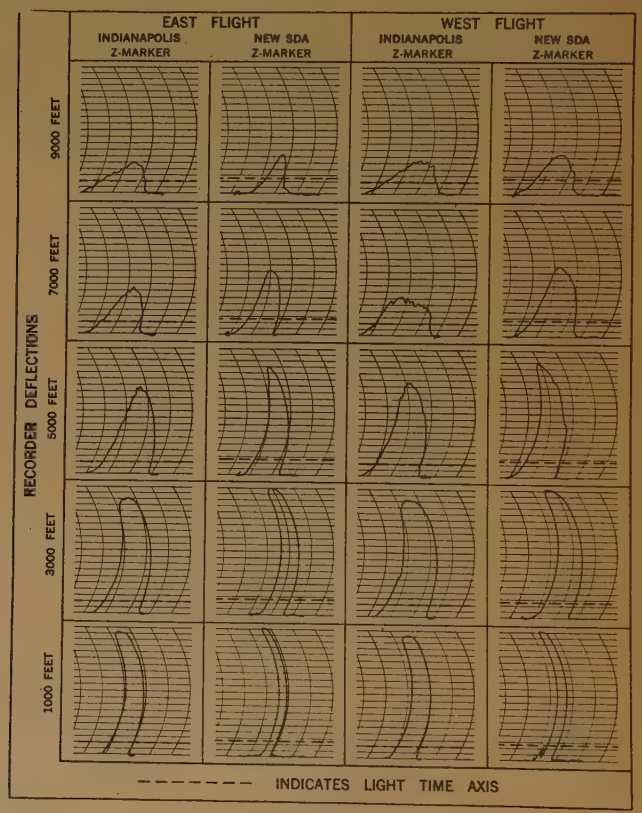


Fig. 11—Copies of actual flight recordings of the present Z-marker and the new spaced-dipole array taken at various altitudes showing comparisons of both arrays under identical flight conditions.

from the chart recording, which moves through the recorder at a constant speed. The value of rectified voltage at which the time axis was established is equal to the maximum value of rectified voltage recorded at the top of the signal zone, namely, 10,000 feet. The width of the signal zone as measured in time was converted to distance in feet from readings of the indicated air speed, taking into account the proper correction factors for temperature, altitude, wind direction, and wind velocity.

Sample recordings of the signal zones of both Z markers for altitudes from 1000 to 9000 feet are shown in Fig. 11.

The height of the signal zone was obtained from recorder deflections taken on flights over the marker at two or more altitudes which are sufficient to insure linear

[5] Manufactured versions of marker antenna systems employ a galvanized-steel angle framework, although a wood frame was used in the test model. Since this work was completed, a standard dipole has been developed for use in marker antennas. The new dipole has 2-inch diameter horizontal members, and the pedestal and fittings are of standard iron conduit and condulet fitting construction. A heavy water spray over the insulators causes less than 2 per cent change in field strength at 100 feet distance. Likewise, the use of solid dielectric flexible cables instead of the $\frac{1}{4}$-inch dual-shielded transmission line and gas-sealed fittings is contemplated. The objective is to prefabricate dipoles and transmission-line sections in the factory for standardization and easy field assembly.

operation of the receiver. The maxima of the deflections were then plotted for various altitudes and a straight line was drawn through them. The altitude at which this line intersects the value used as a light-time axis is the height of the signal zone. Curve *A* of Fig. 12 illustrates such a signal-height determination. This curve intersects the light-time axis at an altitude of 8800 feet. It will be noted that at low altitudes, the region of strong signal strengths, the maximum deflections are limited to eight units by the action of the automatic volume control of the receiver. The straight portion of curve *A* was extrapolated until it intersected the abscissa at zero altitude, point *P*. This method of determination was used to check the height of the signal zone of the spaced-dipole array when the power to the array was increased to give a zone height of 20,000 feet. The airplane used

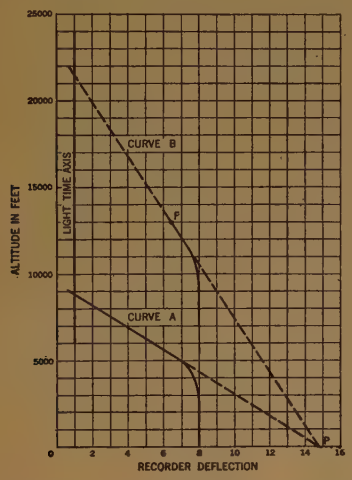
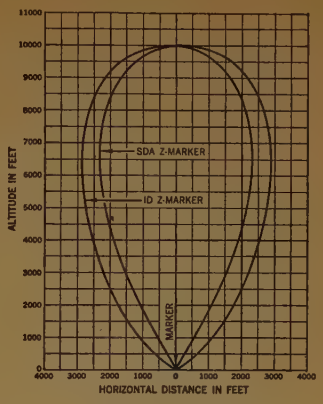


Fig. 12—Signal-zone-height determination.

for the test flights was not capable of flying to 20,000 feet, and it was necessary to resort to this method of extrapolation to determine the zone height. The maximum deflection of the recorder was determined for a test flight on the spaced-dipole signal zone at an altitude (13,000 feet) sufficient to insure linear operation of the receiver. This deflection was plotted on Fig. 12, point *F*. A straight line was drawn through points *P* and *F* (curve *B*) and extended until it intersected the light-time axis. The altitude at this point of intersection, 21,500 feet, was the extrapolated height of the spaced-dipole signal zone. Other values of recorder deflection versus altitude were determined by flight test on this signal zone up to 13,000 feet, and the curve is shown in solid line from point *F* to the limiting abscissa of recorder deflection. For all values of recorder deflection, which are directly proportional to field strengths, the ratio of altitudes for the straight portions of curves *A* and *B* is constant, thus substantiating the inverse law relating the attenuation of the electric field with distance along a constant elevation angle.

The signal zones received from the spaced-dipole marker and the Indianapolis marker for altitudes up to 10,000 feet above ground are shown in Fig. 13. These patterns were obtained under conditions of negligible wind. Flights were made in the east-west direction over both markers. The ratios of the widths of the patterns at several altitudes are tabulated in Fig. 13.



WIDTH OF SDA Z-MARKER COMPARED TO ID Z-MARKER IN PERCENT

ALTITUDE	PERCENT
7000	78.5
6000	78.5
5000	77.0
4000	74.0
3000	65.5
2000	59.0
1000	50.0
500	46.0

Fig. 13—Z-marker patterns—measured zone widths for 10,000-foot signals.

Fig. 14 shows the same patterns as Fig. 13, except that the pattern for the spaced-dipole marker has been extended to 20,000 feet above ground, that is, twice the height of the Indianapolis marker. The current in the dipoles of the spaced-dipole antenna was raised to twice

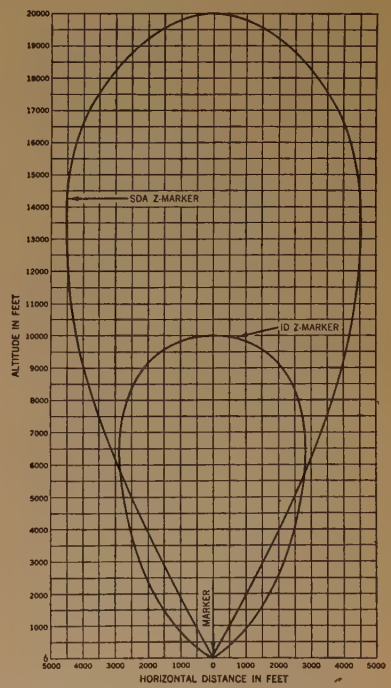


Fig. 14—Z-marker patterns—measured zone widths for 10,000- and 20,000-foot signals.

the value which existed at the time the data for Fig. 13 were obtained. Test flights were then made on both Z markers up to and including an altitude of 13,000 feet. These test flights confirmed the data plotted in Fig. 14.

From these patterns it can be seen that the spaced-dipole marker zone is narrower than the present Z-marker zone below 5700 feet, even though its maximum height is twice that of the present Z marker. The ratios of zone widths up to 5000 feet are tabulated in Fig. 14.

The increase in the widths of the signal zones of both markers for parallel flights to each side of the marker antennas at an altitude of 500 feet above ground is

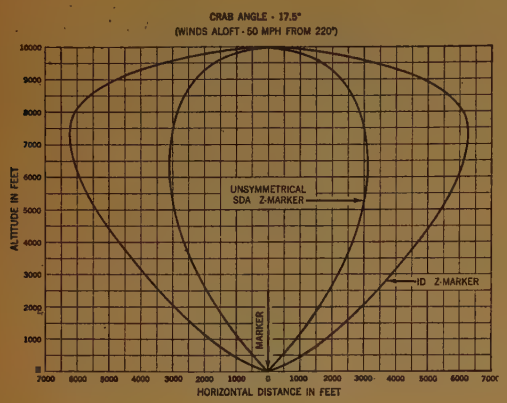


Fig. 15—Z-marker patterns under flight condition of large crab angle

shown in Fig. 6. In these two tests the receiver sensitivity was adjusted to give a signal zone height of 10,000 feet. The elongated pattern for the spaced-dipole Z marker is also shown for an altitude of 3000 feet. The parallel-chord flights on the Indianapolis Z marker were not made in a truly east-west direction since the dipoles of this marker are aligned 30 degrees clockwise from the true north-south, east-west direction.

During these flights it was discovered that the aircraft receiving antenna had a greater pickup on the right side than on the left side of the airplane for the same angle of reception with respect to a vertical plane through the receiving antenna. Therefore, it was necessary to average the data obtained and to compensate for this difference in pickup as well as for differences in signal widths computed for flights in opposite directions. The composite averages for both the spaced-dipole and the Indianapolis markers are shown in Fig. 6. The area within the patterns represents the horizontal area at an altitude of 500 feet over which a signal equal to or greater than a value sufficient to light the signal lamp will be received for parallel flights to either side of the station and perpendicular to the major axes of the elliptical patterns. The average widths of the curves in the major directions are as follows: for the spaced-dipole marker, 1900 feet; for the Indianapolis marker, 2800 feet; a ratio of 0.68. The fact that the ratio of the average distances in the major directions is not so low as the widths of the signal zones at 500 feet, as shown in Fig. 13, can be accounted for by the slightly greater degree of symmetry of the Indianapolis marker array, as explained previously. The ratio of the major to the minor axis at a 500-foot altitude for the spaced-dipole Z marker is 4.75, and for the Indianapolis marker it is 4.5. This is further substantiation of the differences in symmetry.

If the limit of the major axis of the elongated pattern were plotted for various altitudes, it would have the form of the $f_B(\theta)$ curves of Figs. 3 and 4. If the receiving pattern of the airplane antenna were uniform for various angles in a plane normal to the antenna axis, the ratios of the width of the $f_B(\theta)$ curve to the width of the $f_A(\theta)$ curve at various relative altitudes would give a close approximation of the ratios of the major to the minor axes of the elongated patterns at those altitudes. Actually, the ratios as measured are slightly greater than those predictable by the vertical patterns in Figs. 3 and 4. From these tests and calculations made of the airplane receiving patterns it may be concluded that the width of the signal zone and the elongation of the pattern for parallel flights to one side of the Z marker are also functions of the characteristics of the receiving antenna on the airplane.

The results of test flights made during a time of high winds aloft are shown in Fig. 15. These flights were made during the time the unsymmetrical array was being used. The crab angle was 17.5 degrees and the wind velocity averaged 50 miles per hour between 2000 and 10,000 feet above ground. It will be noted that the enlargement of the pattern is more pronounced in the case of the Indianapolis marker than in the case of the spaced-dipole marker. A comparison of Figs. 13 and 15 shows that even under this extreme crab angle the spaced-dipole marker pattern is only slightly larger than the Indianapolis marker pattern shown in Fig. 13, which was obtained under the most favorable wind condi-

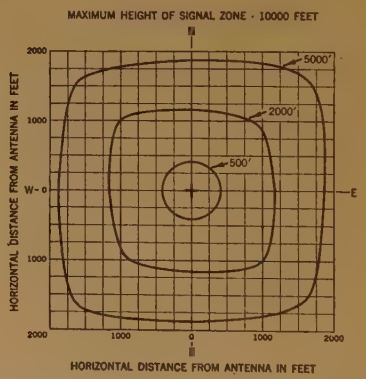


Fig. 16—Horizontal pattern of spaced-dipole Z marker for radial flights at altitudes of 500, 2000, and 5000 feet.

tions. The zone width due to crabbing is increased because of the increased pickup of the receiving antenna as the axis of the antenna is turned away from the direction of flight. It is to be expected that the enlargement of the symmetrical spaced-dipole Z-marker pattern due to crabbing will be slightly greater than that for the unsymmetrical spaced-dipole Z marker, due to its larger $f_B(\theta)$ pattern, but still not as great as that for the Indianapolis Z marker.

The horizontal cross-sectional patterns of radial

flights over the marker are not circular at all altitudes. This may be ascribed to the fact that the received field along different radii over the marker is the vector resultant of the fields from both arrays in quadrature time phase. Horizontal patterns were taken on radial flights at altitudes of 500, 2000, and 5000 feet (Fig. 16). The height of the spaced-dipole marker was 10,000 feet during these tests. The 2000-foot and 5000-foot patterns are almost square. The 500-foot pattern is practically circular.

Fig. 17 shows a plot of the zone widths of the Indianapolis and the new spaced-dipole markers, similar to that illustrated in Fig. 14 except that the light-on period is plotted in terms of time in seconds versus altitude, for 120 miles per hour, ground speed.

Conclusion

It has been shown that the spaced-dipole antenna system possesses a number of distinct advantages over that used with present Z markers and will provide pilots with a more accurate fix because the zone width is considerably less at all altitudes up to 10,000 feet. The marker zone can be extended to 20,000 feet altitude without modification of the present transmitters. Under these conditions the zone width will be less than that of present markers up to 5700 feet. The broad marker zone normally observed on flights off to one side of the marker, or under conditions involving a large crab angle, is appreciably reduced. The radiating elements are of simple and rugged design and will provide stable Z-marker signals under adverse climatic conditions. The dipoles and precut sections of transmission line may be assembled in the factory as prefabricated units ready for field installation.

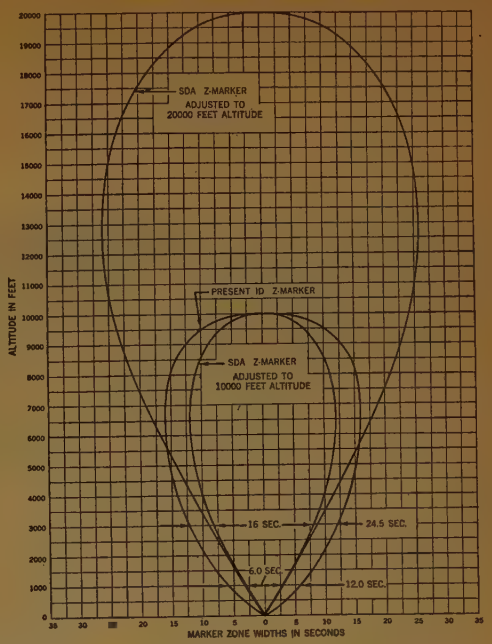


Fig. 17—Z-marker patterns, width in seconds, for 120-mile-per-hour ground speed.

Vacuum Capacitors*

G. H. FLOYD†, Associate, I.R.E.

Summary—**This paper describes the properties, characteristics, and uses of the vacuum capacitor. The constructional details of two General Electric vacuum capacitors, the GL-1L38 and the GL-1L22, are discussed. Design considerations are discussed from the viewpoint of both the designer and the manufacturer. Capacitance formulas are given, and the equation for energy loss is derived. Operating characteristics and ratings of the vacuum capacitor are considered, and the effects of humidity, temperature, and vibration are noted. The advantages of the vacuum capacitor are thoroughly discussed and the applications which are brought about because of these advantages are described.**

I. Introduction

WHEN the word "capacitor" is brought up in a discussion, the average engineer immediately attempts to classify the particular capacitor under discussion in one of four general capacitor groups—paper, mica, electrolytic, or air. It is the purpose of this paper to present some of the details of a rapidly growing newcomer to the capacitor group; namely, the vacuum capacitor. The idea of enclosing capacitor electrodes in an evacuated bulb is not new. It has not been until recently, however, that new application demands and the advantages of newly designed vacuum capacitors caused them to be required in large quantities.

Recent strides in the design of aircraft to operate at high altitudes have opened up many new and interesting problems in the design of radio equipment to accompany these high-flying airplanes. Most of the electronic design problems can be ably solved if the following points are taken into consideration. The points given are only those which arise due to the extraordinary service conditions under which military aircraft operate.

1. Low Temperatures: Temperatures of −50 degrees centigrade are not unusual at altitudes of more than 40,000 feet.

2. High Temperatures: With aircraft operating in all climates, temperatures of 60 degrees centigrade may be experienced.

3. Low Pressures: At 40,000 feet altitude, the pressure is approximately 5.5 inches of mercury.

4. High Humidity: This effect is aggravated by the rapid descent of aircraft into regions of normal pressure,

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temperature, and humidity, as the radio apparatus will still be at very low temperature and condensation of the water vapor present may cover the surface of all equipment.

The vacuum capacitor has many advantages which allow it to operate satisfactorily under the conditions described above.

Two such capacitors, the GL-1L38 and the GL-1L22 are shown in Fig. 1.

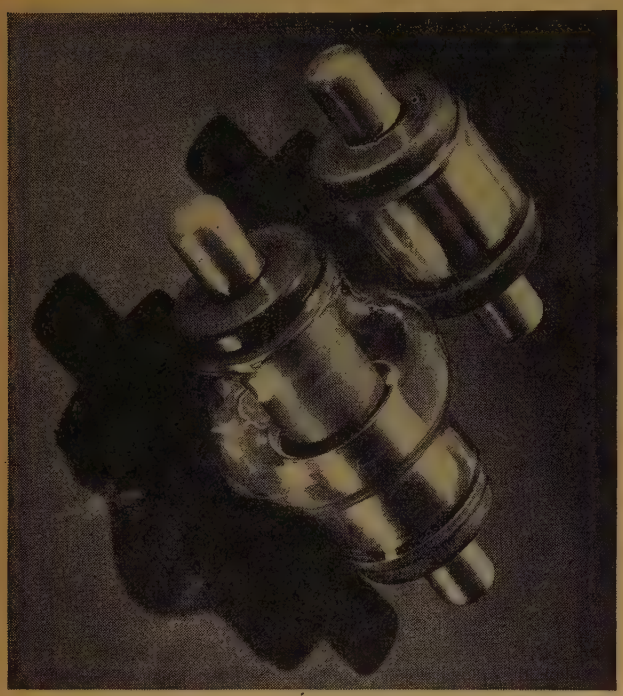

Fig. 1.
Top—GL-1L38 vacuum capacitor.
Bottom—GL-1L22 vacuum capacitor.

II. Constructional Details

Size

The smaller the vacuum capacitor can be made, the more it can be used to advantage. Weight is also a consideration, but this is associated with internal construction as well as with size. The mechanical consideration of size, which is almost entirely a manufacturing problem, is second in importance to electrical considerations.

Electrically, the size will depend upon three factors:

1. Length of the external path required to withstand the rated voltage at the low pressures encountered at high altitudes.
2. Spacing of the internal electrodes required to give the proper capacitance and the rated voltage breakdown.
3. Bulb shape and size required to keep high-loss materials out of the radio-frequency field present between the capacitor electrodes. (In the case of capacitors used for direct-current blocking applications, the last point is not important.)

The first factor, that of the length required for sufficient external breakdown, is important in that it dictates the over-all length of the capacitor. The use of skirts to increase the effective length of path is not practical, as the vacuum enclosure is usually a glass bulb, and adding skirts to a glass bulb would be costly. Furthermore, these skirts would add to the diameter of the capacitor.

The last two factors jointly determine the diameter of the capacitor. The internal spacing required will depend upon the geometry of the electrodes and the capacitance required. The power factor of the bulb used will determine the distance required between the internal electrodes and the bulb to avoid excessive dielectric loss due to the radio-frequency field.

Evacuated Envelope

Fig. 2 shows the two types of evacuated envelopes in

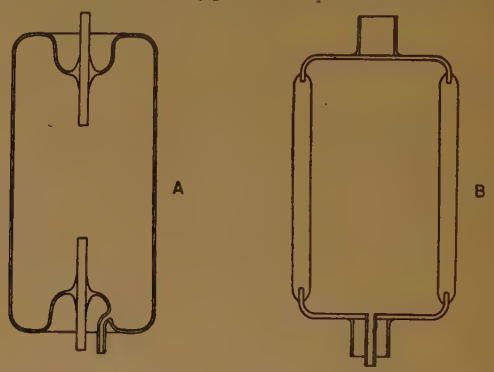

Fig. 2—Two types of evacuated envelopes for vacuum capacitors.

use for vacuum capacitors. Type *A* (left figure) makes use of the flare-to-bulb type of seal, and type *B* (right figure) employs the fernico-to-glass, edge-type seal. Each type has its advantages and disadvantages. Type *A* may make use of any combination of metal and glass whose coefficients of expansion match well enough to allow a seal to be made. Several leads may be run through the press, allowing for a variety of internal mounting schemes. The choice of glass allows the use of one which has a low power factor. Type *A* is not mechanically strong since it is inherent in this design that the stress of mounting is either on the leads themselves or on the glass envelope. Also, the leads do not lend themselves readily to simple mounting of the internal elements by mass-production methods. If concentric electrodes are used, the leads must be perfectly straight for precise mounting.

Type *B* uses end cups of fernico, an alloy of iron, nickel, and cobalt and a borosilicate glass with the same expansion coefficient. Other metal-glass combinations may of course be used. The edge-type seal provides a strong mechanical joint. As may be seen in Fig. 2, the fernico is embedded in the glass cylinder, and the glass makes a vacuum-tight seal on both the inside and outside of the fernico cup. The ferrule which is welded in the center of the fernico cup on the outside provides an axial support which is readily employed in the accurate line-up of the internal elements. A metal exhaust tube, seen on the lower end of type *B* is brazed to the fernico cup.

The main disadvantages of type *B* are that the fernico cup presents a high-loss material to the high-frequency field, and the necessarily heavy glass adds slightly to the loss because of its thickness.

Internal Construction

The space inside a cylindrical bulb is best utilized when the coaxial cylinder type of capacitor is employed. A cross section of a GL-1L38 vacuum capacitor using this type of construction is shown in Fig. 3. The GL-1L38 is rated at 50 micromicrofarads and 7500 volts peak. The over-all length is $3\frac{5}{16}$ inches, and the diameter is $1\frac{5}{8}$ inches.

The coaxial cylinders are shown at *A*. The ends of the cylinders are spun over towards each other for ease in welding. The copper cylinders are welded between a ring of nickel *B* and a steel backup plate *C*. This backup plate provides a strong support for the cylinders, and lessens the possibility of the cylinders deforming under heat or vibration. It also permits the use of a thin end cup of fernico.

D, *E*, and *F* are, respectively, the exhaust tubulation, the support ferrule, and the fernico end cup.

The glass-to-metal seal is made by lining up the upper and lower assemblies in a vertical machine which accurately aligns the two assemblies on the center line of the ferrules. The glass bushing *G* is held in place by a centering clamp, and the upper and lower seal made simultaneously by high-frequency induction heating. As the seal is made, the hot fernico cups are pressed into the glass bushing a predetermined distance.

After exhaust and subsequent processing, the silver-plated-copper end caps *H* are soldered on the ferrules. The size of these caps is such that the whole unit may be mounted in standard-size fuse clamps.

The problem of selecting metals for use as cylinders in the vacuum capacitor involves the same considerations as the choice of any metal for use inside a vacuum tube. Availability, ease of forming, freedom from gas, melting point, and impedance to high-frequency current are, in general, the main factors to consider.

The thickness of the coaxial cylinders is limited on the one hand by the strength required. Cylinders which are too thin are difficult to mount. On the other hand, increasing the thickness of the cylinders cuts down the spacing, decreases the breakdown voltage, and increases the capacitance. A thickness of 0.010 inch has been found satisfactory in the GL-1L38.

III. Design Considerations

Capacitance Calculations

The formula for the calculation of the capacitance of concentric coaxial cylinders is expressed as

$$C = 0.2416(L)/\log_{10} D_1/D_2 \qquad (1)$$

where

C = the capacitance in micromicrofarads
L = the overlap of the two cylinders in centimeters
D_1 = the inside diameter of the outer cylinder in centimeters
D_2 = the outside diameter of the inner cylinder in centimeters

Equation (1) will give the capacitance of a pair of cylinders. To calculate the capacitance for a set of coaxial cylinders, it is necessary to calculate the capacitance for each pair of cylinders and add the respective results obtained.

The result will not be the exact capacitance of the finished capacitor, as the formula neglects end effect. In

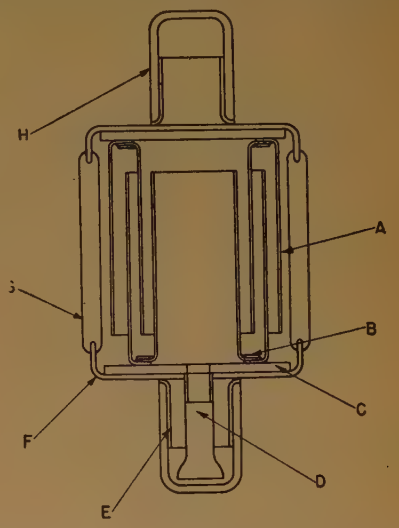

Fig. 3—Cross section of GL-1L38 vacuum capacitor.

general, the extra capacitance due to end effect will depend upon 1. number of cylinders, 2. thickness of cylinders, 3. length of cylinders, and 4. spacing between end of cylinders and opposing cylinder supports.

The importance of the end effect in the over-all calculations will be governed entirely by the construction of the capacitor. In the GL-1L38, the end effect causes an increase of approximately 6 micromicrofarads over the calculated capacitance. In this case, the end effect amounts to more than 10 per cent of the total capacitance.

Equation (1) assumes that the cylinders are perfectly coaxial. Any variation from this condition will change the capacitance. A variation in spacing due to axial displacement of the cylinders increases the capacitance, but the change is not large until the two cylinders are almost touching. This is one advantage which the coaxial type of construction enjoys over the flat-plate type of capacitor construction.

Spacing

Under perfect conditions, a spacing of 0.050 inch between two electrodes in a perfect vacuum should hold off voltages of 50 to 75 kilovolts. The exact hold-off voltage would depend upon the type of electrodes used. In the manufacture of vacuum capacitors it is virtually impossible to achieve these perfect conditions. Inasmuch as the breakdown voltage depends upon several

factors, it is necessary to investigate the relative importance of these factors. Other than the spacing factor, the important factor is the degree of vacuum obtainable in the envelope. The General Electric vacuum capacitors are processed to a degree of vacuum which will ensure stable operation without being affected by ionization of residual gas. As a further precaution, smooth and well-rounded surfaces on the metal are desirable to prevent any sharp breaks in the high-voltage field.

The major problem in the manufacture of vacuum capacitors is undoubtedly the assembly of the coaxial

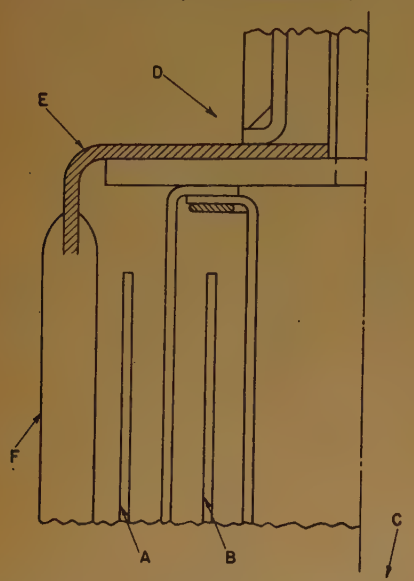

Fig. 4—Enlarged cross section of hot end of GL-1L38.

cylinders. A generous allowance for misalignment must be made. This allowance, in most cases, is large enough to overshadow the effect of the other breakdown voltage factors.

Energy Losses

The losses in vacuum capacitors of the type under discussion may be attributed mainly to two sources:

1. Resistance loss in the metallic structure.
2. Ionic loss in the material subjected to the high-frequency field.

The resistance loss at frequencies up to 50 megacycles may be considered as negligible. The construction of a vacuum capacitor of the GL-1L38 type is such that the cross-sectional area of conducting surfaces is large. There are no wire leads between which the current must divide. In this way the resistance is kept to a minimum.

The ionic loss in the material subjected to the high-frequency field presents the main design problem. The care with which this problem is handled will determine the maximum frequency and voltage limits of the vacuum capacitor.

Extensive tests have shown that the major part of the loss of the vacuum capacitor occurs in the glass envelope.

Fig. 4 shows a vacuum capacitor of the GL-1L38 type with one quarter in cross section. Coaxial cylinders A and B are connected to end C. The full voltage across the capacitor, therefore, appears between the ends of cylinders A and B and end D. The voltage stress is particularly high between cylinder A and the junction of the fernico E to the glass F. The glass in that portion of the capacitor is, therefore, subjected to a very strong high-frequency field.

In order to make a seal to the fernico end, a borosilicate glass must be used. The loss that takes place in this glass is caused by the mobility of the alkali ions present. The high-frequency field induces a voltage in the glass, and the current that results is carried by these ions. The accompanying effect is the heating of the glass due to the passage of current.

Failure occurs when the heat generated by the ionic loss can no longer be radiated or conducted away from this band of glass. When this occurs, the heat begins to raise the temperature of the glass in the portion of the envelope that is closest to the end of cylinder A. At one point around the circumference of the tube, this effect will be greater than that at any other point. At this point a runaway effect occurs which punctures the glass envelope. In any capacitor which is constructed as shown in Fig. 4 the failure occurs in the section of glass which is immediately adjacent to the fernico head E.

Before further capacitors could be designed intelligently, it became necessary to determine just what factors contributed to this heating. The current and voltage relationships in any capacitor are expressed by

$$V = IZ. \tag{2}$$

The first assumption to be made in the simplification of the formula is that the resistance to radio-frequency current in the circuit is negligible. The reason for making this assumption will be shown later. With the assumption made, the formula becomes

$$V = 1/2\pi fC. \tag{3}$$

Inasmuch as the tests were to be made with vacuum capacitors of one capacitance, the formula becomes

$$V = KI/f. \tag{4}$$

It is obvious from (4) that temperature-rise measurements cannot be made as a function of any one of the three variables while holding the other two variables constant. Before proceeding further it was therefore necessary to make a third and final assumption.

It had been noticed that when capacitors failed due to the puncture of the glass, the end which punctured would be at a very high temperature whereas the opposite end would be only moderately warm. Further tests were conducted under conditions which caused the failure to occur in a matter of seconds. Under these conditions, the end which did not puncture was found to be at room temperature. Tests conducted at various currents and voltages and at different frequencies showed that regardless of the current, this one end remained at room temperature. It was only when the puncture took a long time to occur that the cold end was raised in temperature. The conclusion reached was that the circuit

current had no effect in producing heat, and that the cold end became heated only when the time of puncturing was sufficiently long to allow heat to be conducted from the hot end to the cold end. This was a logical conclusion, as any heat due to resistance must of necessity have been produced uniformly throughout the length of the capacitor.

Therefore, the assumption was made, that at the frequencies at which the tests were made (1 to 30 megacycles) *the temperature rise was independent of the capacitor circuit current.*

It now became possible to run tests holding one variable constant, and taking readings of the second variable versus temperature rise. The third variable, current, had to vary in accordance with (4), but its effect could be neglected.

A method had to be found by which it would be possible to measure the temperature of the glass without disturbing the results of the test. Some means had to be used, therefore, to measure the temperature when the voltage was removed from the capacitor. This required the use of a device which would come to temperature quickly. Thermometers were not used for this reason.

The solution was found by employing a thermocouple. The junction of the thermocouple was attached to a hinged arm. During the time the voltage was applied to the capacitor, this arm was held in position away from the glass. As a temperature reading could be taken in five seconds after the removal of voltage, there was no appreciable heat loss from the glass in the time required to get this reading.

For the purpose of the tests, the capacitors were run at one voltage and frequency until a stable temperature

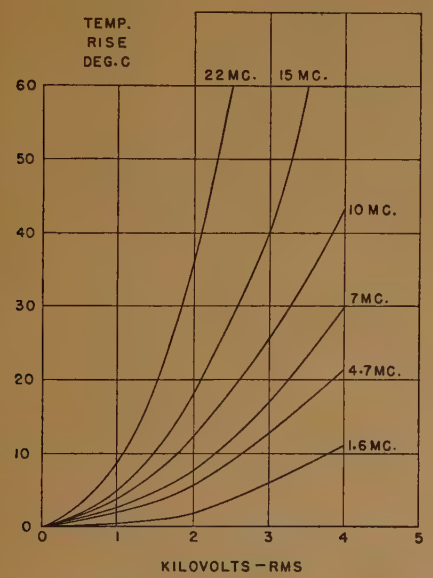

Fig. 5—Vacuum-capacitor temperature-rise characteristics —constant-frequency curve.

was reached. The holder for the thermocouple junction was made from a heat-resistant material. It was shaped so that the thermocouple junction was covered on all sides except where it came in contact with the capacitor. The hinge was arranged so that the junction touched the glass on the capacitor at precisely the same point for each temperature measurement.

Fig. 5 shows the curves of temperature rise in degrees centigrade versus kilovolts. The temperature rise is the

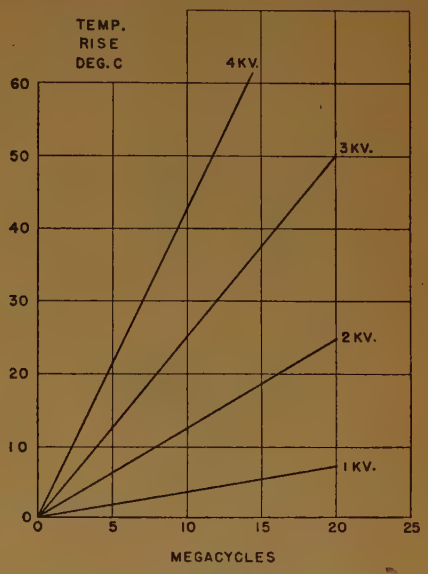

Fig. 6—Vacuum-capacitor temperature-rise characteristics —constant-voltage curve.

rise over ambient temperature experienced by the glass on the hot end of the capacitor. The voltage is the root-mean-square radio-frequency voltage applied across the capacitor. These curves approximate the square law curve, or

$$T = K'V^2. \tag{5}$$

In (5) the constant K' is a constant that changes with the frequency. By solving for this constant K' it can be shown that it varies as

$$K' = K''f. \tag{6}$$

This relationship may also be checked by plotting temperature rise versus frequency. It is evident from Fig. 6, which shows this curve, that the temperature rise varies directly with the frequency. This enables the final formula to be expressed as

$$T = K'''fV^2 \tag{7}$$

where T = the temperature rise above ambient temperature of the glass on the hot end of the capacitor

K''' = the ionic loss constant for any one type of capacitor

f = the frequency of the voltage applied

V = the voltage across the capacitor.

The only controllable factor in (7) is the constant K'''. It is evident that the chemical composition of the glass used will cause each type of glass to have a different constant. For this reason it is necessary to choose a glass which will contribute a minimum ionic loss.

While conducting the tests on the capacitors, the temperature-rise measurements were made at a point on the

glass nearest the termination of the outer metal cylinders, as this point was found to be the hottest. Following this line of reasoning, capacitors were made up with this outer cylinder of varying lengths. When the outer cylinder was of zero length, the former cold end became the hot end. The temperature rise was not so high as before, as the glass was a greater distance from the new outer cylinder. When the outer cylinder was one half normal length, the temperature rise was greatest in the central portion of the glass envelope. This cylinder length was found to be optimum, as the heat generated had a greater opportunity to be conducted away, and the temperature rise was therefore less than with the full-length cylinder.

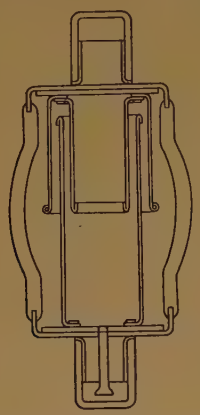

Fig. 7—Cross section of GL-1L22 vacuum capacitor.

These tests gave rise to the new type of design used in the General Electric 16-kilovolt capacitor line. One of these capacitors, the GL-1L22, is shown in cross section in Fig. 7. The GL-1L22 is rated at 25 micromicrofarads at 16 kilovolts peak voltage. The outer cylinder has been made shorter so that it terminates in the center of the capacitor. The glass in the center of the bulb has been bulged out so that a large distance is obtained between the end of the outer cylinder and the inside of the glass. The end of the outer cylinder has been rolled over, further to reduce the electrical stress set up in the glass.

IV. Operating Characteristics and Ratings

Voltage and Current Relationships

In comparison to capacitors subject to atmospheric pressures, the breakdown voltage of a vacuum capacitor is relatively more constant throughout life. There may be an occasional minute quantity of gas released internally, due to overload or voltage surge. A voltage surge above the rating of the capacitor does not puncture the dielectric, and the tendency is for the capacitor to clean up. The breakdown voltage, therefore, normally reverts to its original value. In operation, there is no dielectric to deteriorate with time.

The current ratings of the General Electric vacuum capacitors are obtained indirectly from (7). As it is desirable to use the vacuum capacitors under varying conditions of voltage and frequency, it is impossible to rate the capacitors correctly by stating only a maximum current. The criterion of maximum operation is the energy loss. The ratings used, therefore, are those which will cause the vacuum capacitor to operate at some given maximum temperature when operated at maximum ratings. By the same reasoning as that applied to current, it is impossible to give only a maximum frequency limit.

The following is an example of the current, voltage and frequency relationships. A GL-1L38 will have a temperature rise of approximately 40 degrees centigrade when operated at 6 megacycles, 10 amperes root-mean-square current, and 7.5 kilovolts peak voltage. By the use of (7) the voltage at 10 megacycles may be computed which will give the same temperature rise. This voltage is 5.8 kilovolts, and the corresponding current is 12.6 amperes root-mean-square. At 40 megacycles, the voltage is 2.9 kilovolts and the current is 25 amperes root-mean-square.

Temperature Coefficient

The expansion and contraction due to temperature changes of the glass and metal parts in the types of capacitors under discussion are a complex movement. As the capacitor becomes heated, the following changes take place:

1. The glass envelope tends to lengthen along its axis, causing a decrease in capacitance.
2. The glass envelope tends to enlarge in diameter, placing a bending stress on the fernico end cup, and tending to displace the cylinders on their axes.
3. The fernico end cup tends to enlarge its diameter, at a different expansion rate from that of the glass envelope, adding its effect to that of (2) above.

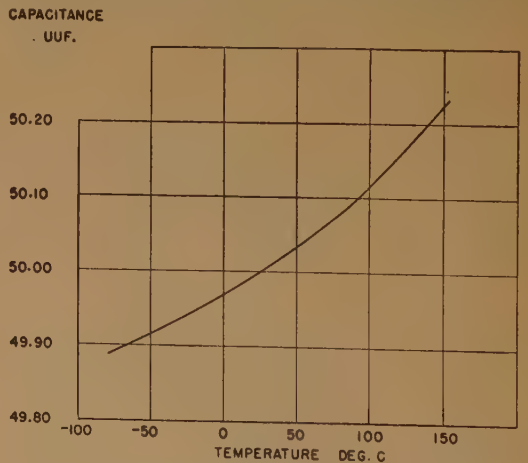

Fig. 8—Temperature coefficient curve for GL-1L38 vacuum capacitor.

4. The copper cylinders tend to lengthen along their axes, causing an increase in the capacitance.
5. The copper cylinders tend to enlarge their diameter, causing the capacitance either to increase or decrease, depending upon which cylinder is considered.

Fig. 8 shows a curve of capacitance versus temperature for the GL-1L38. The curve is that of one particular capacitor. The temperature coefficient for that

particular capacitor, computed on the part of the curve from −50 degrees centigrade to +100 degrees centigrade, is 27.0×10^{-6} micromicrofarad per micromicrofarad per degree centigrade.

The coefficients of expansion of the various materials used in the GL-1L38 would cause a much greater change in capacitance if figured individually, but we find that the interactions between the materials compensate for one another to produce a very satisfactory temperature coefficient.

Humidity and Vibration Effects

Changes in humidity have no effect upon the capacitance of the vacuum capacitors. As moisture can affect only the outside surface of the capacitors, high humidity may cause a slight leakage current across the outside of the capacitor. Specimens cooled to approximately −30 degrees centigrade, and then placed in a high-humidity atmosphere were covered with condensed droplets of water. This moisture affected the creepage breakdown potential on the outside of the capacitor momentarily, but the surface dried in a few seconds and the external breakdown potential returned to normal. This drying takes place best when the capacitor is operating at somewhere near normal ratings.

In the case of a capacitor operating in an atmosphere of high humidity, where there are no temperature changes to cause sudden condensation, the decrease in creepage breakdown potential is less noticeable than in the case above. Under any conditions of high humidity, the breakdown path on the GL-1L38 and similar capacitors is still sufficiently long to ensure that the breakdown voltage is greater than the capacitor ratings, even at altitudes up to 30,000 feet.

Vibration does not affect a cylindrical-type capacitor in the same manner as it does a flat-plate capacitor. In the latter type, displacement of the plates by vibration is accompanied by a capacitance change which is in direct ratio to the displacement. Such is not the case in cylindrical-construction vacuum capacitors.

Fig. 9 shows the relationship between capacitance change and axial displacement. This curve is a hyperbolic cosine function, and may be computed from

$$C = 1/2\left[\cosh^{-1}\left\{-(D^2 - R_1^2 - R_2^2)/(2R_1R_2)\right\}\right]^{-1} \quad (9)$$

where

C = the capacitance in micromicrofarads per centimeter of overlap

D = the axial displacement of the cylinders in centimeters

R_1 = the radius of the outer cylinder in centimeters

R_2 = the radius of the inner cylinder in centimeters

This formula[1] is for the case where the two cylinders are contained one within the other.

Fig. 9 is plotted for the General Electric GL-1L38 capacitor. An axial displacement of 0.010 inch, which is a displacement of approximately 20 per cent of the spacing, causes a change in capacitance from 50.0 micromicrofarads to 50.7 micromicrofarads, or less than one and one half per cent of the total capacitance.

The usual displacement caused by vibration is not an axial displacement, as the axes of the concentric cylinders are not free to move. Vibration causes a motion which tends to bend the cylinders, the pivot point being

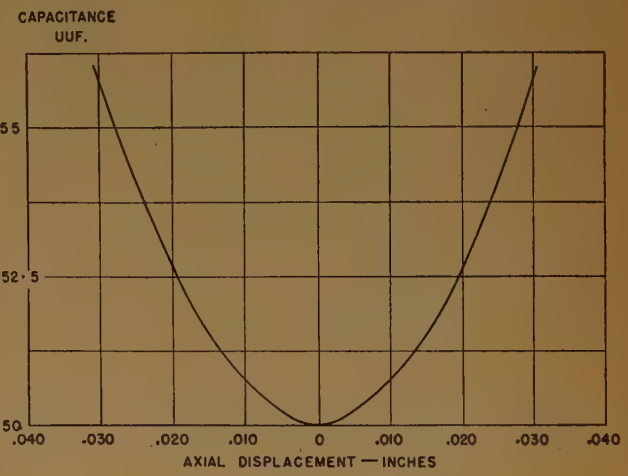

Fig. 9—Axial-displacement—capacitance-change curve for concentric cylinders.

the point where the cylinders are attached to their end plates. This motion has an effect on the capacitance which is even less than the effect produced by the axes being displaced. For these reasons, the vacuum capacitor is affected a minimum amount by vibrations.

V. Advantages

Due to the Vacuum

As would be expected, most of the advantages of vacuum capacitors stem from the fact that the dielectric employed is a vacuum. The main advantages gained from this are:

1. Stable Dielectric Constant: Unlike any other form of capacitor, the vacuum capacitor has a dielectric constant whose value is unity. This value stays constant regardless of temperature change or humidity change. Changes in capacitance due to temperature change result entirely from the fact that the mechanical structure changes size, and as shown in Fig. 9 these changes are very small. Capacitance change due to temperature change of the dielectric does not exist.

2. Stable Internal Breakdown Voltage: This is one of the very important advantages of the vacuum capacitor. Humidity cannot be present to cause lowering of the breakdown voltage. Low air pressures, such as those found at high altitudes, do not change the breakdown voltage.

3. Low Loss: The fact that all capacitors, with the exception of air and vacuum capacitors, use some material as a dielectric which contributes a dielectric loss, means that inherently vacuum capacitors are lower-loss

[1] William R. Smythe, "Static and Dynamic Electricity," McGraw-Hill Book Company, New York, N. Y., 1939.

devices than capacitors using some material as a dielectric.

4. Ability to Withstand Overvoltages: The vacuum capacitor does not depend upon a solid dielectric for its voltage insulation. For that reason, there is no dielectric to puncture if overvoltages are applied to the capacitor. On either direct current or radio frequencies, overvoltages may cause a discharge to take place internally. In most of these cases, this discharge is not injurious to the vacuum capacitor, and when the overvoltage is removed, the capacitor will function as usual. Overvoltages may harm the capacitor if there is sufficient power in the source. The heat generated by the arc in this case may be enough to melt the internal electrodes.

5. Maintenance: A minimum of maintenance is required on vacuum capacitors. Dirt and dust cannot affect the internal structure of the vacuum capacitor. Externally, it takes a great deal of contamination to lower the breakdown voltage to a point where cleaning is required.

Advantages Derived from the Mechanical Construction

1. Size: In comparison to capacitors which use a high dielectric constant, vacuum capacitors are not small. However, in comparison with high-voltage, high-frequency capacitors, vacuum capacitors are very compact. Their compact size allows them to be used with advantage in high-frequency circuits where long leads and bulky apparatus cause inefficiency.

2. Sturdiness: Normally, tubes made of glass and metal are not considered durable. The vacuum-capacitor construction produces a tube which is mechanically strong. Such vacuum capacitors have been dropped from heights of several feet without sustaining damage. Capacitance is very stable even under conditions of heavy vibration.

The vacuum capacitor is also very strong thermally. General Electric vacuum capacitors may be cooled to −50 degrees centigrade and immersed immediately in water at 100 degrees centigrade without harm.

3. Interchangeability: These types of vacuum capacitors are designed with sturdy terminals which allow them to be plugged in and out of circuits with ease.

VI. Applications

For the most part vacuum capacitors are used in applications where their particular characteristics result in the greatest benefit to the user. Obvious applications of vacuum capacitors, with their constant breakdown voltage, are in aircraft installations. All other conditions of military-aircraft applications are met by vacuum capacitors. Sturdiness, low maintenance cost, and ability to withstand extremes of temperature and humidity make vacuum capacitors ideal for this service.

Interchangeability is advantageous in the design of high-frequency oscillators or amplifiers where flexibility is desirable. In these applications, tuning may be accomplished by variable inductances.

Induction-heating oscillators of the self-excited types require blocking and by-pass capacitors. In high-power units, the current-carrying ability of these capacitors must be large. Vacuum capacitors are able to pass higher currents per unit size than any other type of capacitor, and may be used in parallel quite readily.

Characteristics of Voltage-Multiplying Rectifiers*

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***Summary*—A combined experimental and theoretical analysis is used to determine the manner in which the characteristics of the half-wave and full-wave voltage-doubling rectifier circuits depend upon the resistance of the diodes and upon the load resistance. The characteristic curves reproduced include those for the load voltage, ripple voltage, and maximum tube currents. Since it was found that the rectifier characteristics are nearly independent of the load-resistance parameter in the usual operating range, it is possible to obtain general characteristics which hold not only for the doublers but also for multiplying rectifier circuits of higher orders. Various combinations of these multiplying circuits are also discussed.**

Introduction

THE increase in the use of small vacuum-tube devices and other instruments requiring a direct voltage has created a demand for data suitable for use in the design of the necessary power supplies for such instruments. The present shortage of certain types of transformers brings new attention to voltage-multiplying circuits and their operating characteristics. This paper presents the results of a combined experimental and theoretical analysis to determine some of the more important characteristics of two of the many rectifier circuits using vacuum-type diodes. The operating characteristics were determined in the laboratory, while the limits of the characteristics, as a parameter of the circuits approached zero or infinity, were determined by mathematical means. The results of the study of the limits led to the solution of a general method of determining the limits for many voltage-multiplying circuits, and a way of using these limits as an aid in determining the approximate operating characteristics was found.

Two diode rectifier circuits having capacitance filters have been considered in detail in this analysis. These circuits, the half-wave and the full-wave voltage-

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doubling rectifiers, have a direct terminal voltage of approximately twice the maximum value of the input alternating voltage at no load when the diode resistance drop is low. Roberts[1] and Schade[2] have made a study of some of the characteristics of the full-wave doubling circuit but have not considered the half-wave voltage-doubling circuit. Previous mathematical analyses[3,4] have considered the two doubler circuits with the main simplifying assumption that there was no voltage drop across the tubes when conducting. It was also stated that further work should be done to remove this assumption, and the results of this work are contained in this paper. A complete mathematical analysis of these circuits was not made, because it was found that a combined experimental and mathematical analysis gave the desired results in much less time. This analysis develops characteristics for the output voltage, the maximum tube current, the inverse peak voltage, and the per cent ripple in the load voltage for both circuits.

Doubler Characteristics

Previous mathematical analyses[1,5] of rectifier circuits have shown that the circuit characteristics are dependent upon the parameter ωCR_L, where R_L is the resistance in ohms of the load, C is the capacitance in farads of the condenser across the load, and ω is the product $(2\pi f)$ where f is the frequency of the supply voltage in cycles per second. All characteristics given in this analysis, other than limiting characteristics, are plotted against this load-resistance parameter. The direct-voltage output data are given in the form of the ratio of the direct voltage across the load divided by the maximum value of the alternating input voltage; the maximum tube-current characteristics are given as ratios of the maximum values of the current passing through the tube to the average direct current passing through the tube. The inverse peak voltage has its characteristic shown as the ratio of the inverse peak voltage to the maximum value of the input alternating voltage. The ripple content of the load voltage is plotted as the per cent ratio of the effective output ripple voltage to the average value of the output voltage.

This analysis, like that of Roberts,[1] assumes that the resistance R_r of the diode is constant and the current varies directly with the voltage. While this is not exactly true physically, it is a good approximation over the range of operation for many diodes and is mathematically and experimentally convenient. The parameter for each curve of the characteristic family is the ratio of the load resistance R_L to the diode resistance R_r. Characteristic curves showing $R_L/R_r = \infty$ as a parameter are calculated curves taken from mathematical analyses[3,4] of these circuits assuming no tube drop, i.e., R_r is zero. By similar calculations it is possible to obtain the curves for $R_L/R_r = 0$. For the characteristic curves of the other values of the R_L/R_r parameter ranging between zero and infinity, the end points of the curves at $\omega CR_L = 0$ and $\omega CR_L = \infty$ are easily calculated as shown in Appendix B. The curves in between these end points are then filled in experimentally. The characteristics apply only to circuits having capacitance filter.

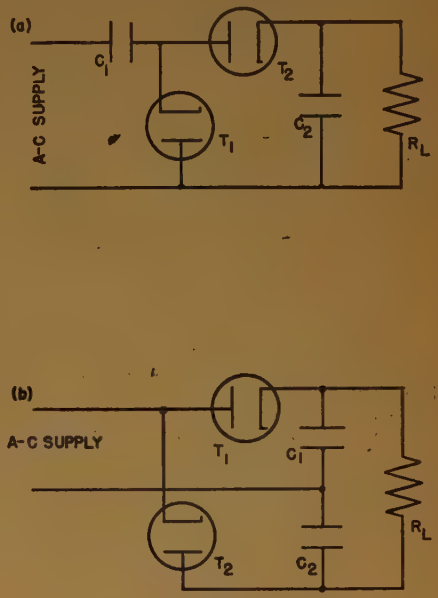

Fig. 1
(a) Diagram of the half-wave voltage-doubling circuit.
(b) Diagram of the full-wave voltage-doubling circuit.

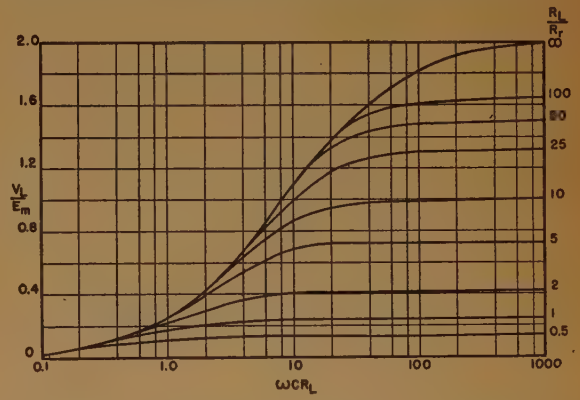

Fig. 2—The load-voltage ratio curves of the half-wave doubler.

The circuit for the half-wave doubler is shown in Fig. 1(a). The half-wave doubler-voltage ratio curves, Fig. 2, show that a voltage multiplication of 1 and greater is not accomplished unless the R_L/R_r ratio is more than ten, and the multiplication does not become appreciable until this ratio becomes fifty or greater. The limits which the voltage curves of this family tend to approach, as ωCR_L approaches zero or infinity, are given in tabular form in Fig. 3. For purposes of comparison the limits of the half-wave and full-wave rectifier

[1] N. H. Roberts, "The diode as a half-wave, full-wave and voltage doubling rectifier," *Wireless Eng.*, vol. 13, pp. 351–362, July, 1936; and pp. 423–470; August, 1936.
[2] O. H. Schade, "Analysis of rectifier operation," Proc I.R.E., vol. 31, pp. 341–361; July, 1943.
[3] D. L. Waidelich, "The full-wave voltage-doubling rectifier circuit," Proc. I.R.E., vol. 29, pp. 554–558; October, 1941.
[4] D. L. Waidelich and C. H. Gleason, "The half-wave voltage-doubling rectifier circuit," Proc. I.R.E., vol. 30, pp. 535–541; December, 1942.
[5] D. L. Waidelich, "Diode rectifying circuits with capacitance filters," *Trans. A.I.E.E.* (*Elec. Eng.*, December, 1941), vol. 60, pp. 1161–1167; December, 1941.

circuits with capacitance filters are also given. The equations for the end points of the voltage characteristics have been solved in Appendix B, and the calculated results are shown in the form of curves in Fig. 4. The characteristics of Fig. 2 are somewhat indicative of the voltage regulation of the circuit because an increase in the load is represented by a decrease in the load resistance. The maximum tube current ratio curves are shown in Fig. 5 and the inverse peak-voltage ratio characteristics in Fig. 6. The ripple characteristics for the half-wave doubler are given in Fig. 7. The limits that these curves approach as ωCR_L approaches zero and infinity are tabulated in Fig. 3.

LIMITING FUNCTIONS FOR CHARACTERISTICS				
ωCR_L	0	∞	0	∞
	Half-Wave Rectifier		Full-Wave Rectifier	
$\frac{V_L}{E_m}$	$\frac{R_L/R_r}{\pi[(R_L/R_r)+1]}$	$\frac{R_L}{R_r}=f\left(\frac{V_L}{E_m}\right)$	$\frac{2(R_L/R_r)}{\pi[(R_L/R_r)+1]}$	$\frac{R_L}{R_r}=\frac{1}{2}f\left(\frac{V_L}{E_m}\right)$
$\frac{i_m}{I_r}$	π	$\frac{R_L}{R_r}\left[\frac{1}{(V_L/E_m)}-1\right]$	π	$\frac{2R_L}{R_r}\left[\frac{1}{(V_L/E_m)}-1\right]$
$\frac{E_p}{E_m}$	1	$1+\frac{V_L}{E_m}$	$1+\frac{1}{(R_r/R_L)+1}$	$1+\frac{V_L}{E_m}$
r	121.2	0	48.3	0
	Half-Wave Doubler		Full-Wave Doubler	
$\frac{V_L}{E_m}$	0	$\frac{1}{2}\frac{R_L}{R_r}=f\left(\frac{1}{2}\frac{V_L}{E_m}\right)$	0	$\frac{1}{2}\frac{R_L}{R_r}=f\left(\frac{1}{2}\frac{V_L}{E_m}\right)$
$\frac{i_m}{I_r}$	π	$\left(\frac{1}{2}\frac{R_L}{R_r}\right)\left(\frac{2E_m}{V_L}-1\right)$	π	$\left(\frac{1}{2}\frac{R_L}{R_r}\right)\left(\frac{2E_m}{V_L}-1\right)$
$\frac{E_p}{E_m}$	0	$1+\frac{1}{2}\frac{V_L}{E_m}$	0	$1+\frac{1}{2}\frac{V_L}{E_m}$
r	121.2	0	48.3	0
Where $f\left(\frac{V_L}{E_m}\right)=\frac{2\pi(V_L/E_m)}{2\sqrt{1-(V_L/E_m)^2}-(V_L/E_m)[\pi-2\sin^{-1}(V_L/E_m)]}$				

Fig. 3—The limiting functions for the various rectifier characteristics.

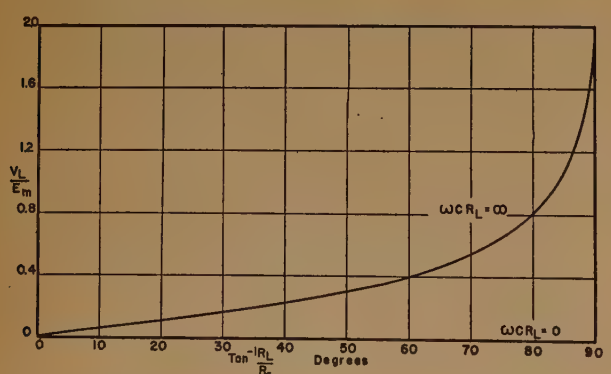

Fig. 4—The load-voltage ratio curves of either doubler for limiting values of ωCR_L.

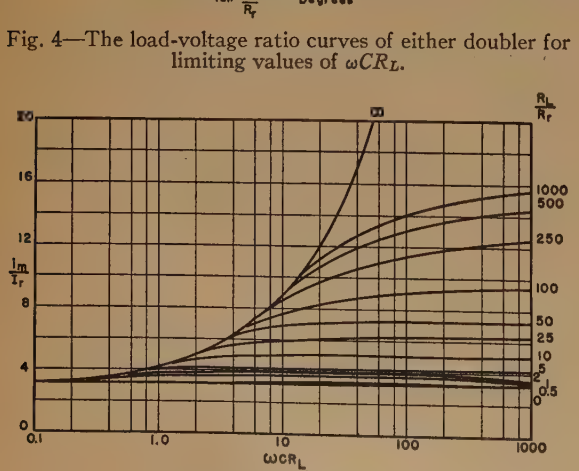

Fig. 5—The maximum tube-current ratio curves of the half-wave doubler.

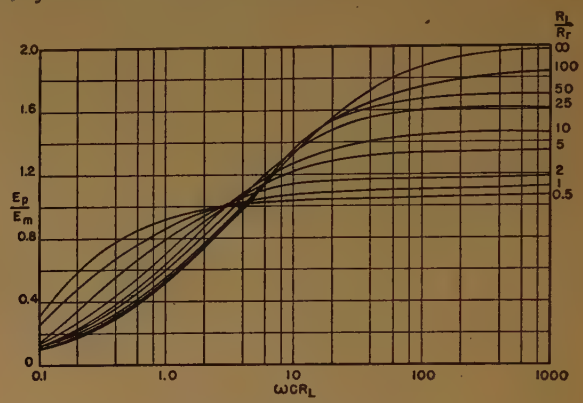

Fig. 6—The inverse peak-voltage ratio curves of the half-wave doubler.

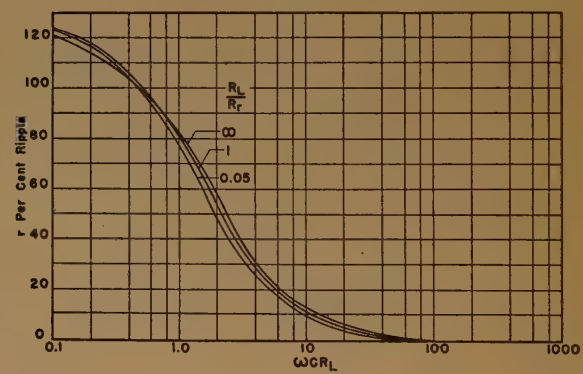

Fig. 7—The per cent ripple curves of the half-wave doubler.

The maximum tube-current characteristics for the half-wave doubler are from data taken for both tubes. In general, T_1 conducts higher current peaks than does T_2. When the peaks are approximately equal in both tubes, the maximum shifts erratically between the tubes. The characteristics shown were determined from the greater of the two current peaks. The inverse peak-voltage ratio curves are from data taken for T_1. In general, the inverse peak voltage appearing across T_1 is greater than that across T_2; however, for light loads the two inverse peak voltages are substantially equal.

A schematic diagram of the full-wave doubler circuit is shown in Fig. 1(b). The characteristics of Fig. 8 show that a voltage multiplication of one and greater is obtained only when the R_L/R_r ratio is 10 or greater. Fig. 9 shows the maximum tube-current characteristics; Fig. 10, the inverse peak-voltage curves; and Fig. 11, the percentage ripple in the output voltage. The curves, with the exception of the ripple characteristics, approach the same limits as do the half-wave doubler characteristics.

Neither of the doubler circuits shows a multiplication greater than 1 when the ratio of the load resistance to the tube resistance is less than 10. The characteristics of the two circuits are very similar; the full-wave doubler showing better regulation and stability than the half-wave circuit. For values of ωCR_L less than 100, the voltage ratios of the full-wave doubler are always larger than those of the half-wave doubler. The maximum tube-current characteristics of the half-wave doubler have larger values in general than those of the full-wave circuit, although both sets approach the same limit for decreasing values of ωCR_L. The inverse peak voltages of the full-wave doubler are higher than those of the half-wave doubler for the most part. As ωCR_L increases, the inverse peak-voltage curves approach values dependent upon the V_L/E_m ratio of the respective circuits. The theoretical maximum ripple occurs for ωCR_L equal to zero and is 121.2 per cent for the half-wave doubler and 48.3 per cent for the full-wave doubler. When both ωCR_L and R_L/R_r are greater than 10, the ripple r is very nearly equal to $(109/\omega CR_L)$ for both circuits. The shapes of the ripple curves are similar, and all approach zero as ωCR_L approaches infinity. For low values of ωCR_L the ripple of the half-wave doubler is approximately twice that of the full-wave doubler for a given value of ωCR_L and has a fundamental ripple frequency of one half that of the full-wave doubler. For either circuit the use of a diode having a greater resistance or the addition of series resistance in the diode circuit does not materially reduce the amount of ripple except for high values of ωCR_L.

Method of Analysis

The data were obtained in a manner similar to the method of Roberts.[1] A type 83 mercury-vapor diode with a resistance in series with its cathode was used to simulate the vacuum diode because this allowed a means of varying the diode resistance and made the test circuits very flexible. The limits approached by the characteristics as ωCR_L approaches zero and infinity are determined by the artifice that the capacitance is zero at $\omega CR_L = 0$ and is infinite at $\omega CR_L = \infty$. An equivalent circuit for the first condition shows no capacitances. The second condition, that the capacitance be infinite, is to say that it is so large that, once charged, it maintains a constant voltage across its terminals and the equivalent circuit may show a source of constant voltage in the position of the condenser. The curves of the end points for the two circuits considered are given in Fig. 4; they are plotted against $\tan^{-1}(R_L/R_r)$ in degrees. The calculated end points were compared with the experimental data, and it was found that the error varied from 2 to 6 per cent for most of the different circuits and characteristics.

The calcuations were performed by writing the Kirchhoff equations for the circuit under examination according to the proper equivalent circuit and solving. Details of the method are given in Appendix B. The method of letting the condensers be of zero capacitance to determine the end points does not serve to give a solution for the full-wave doubler or more complex

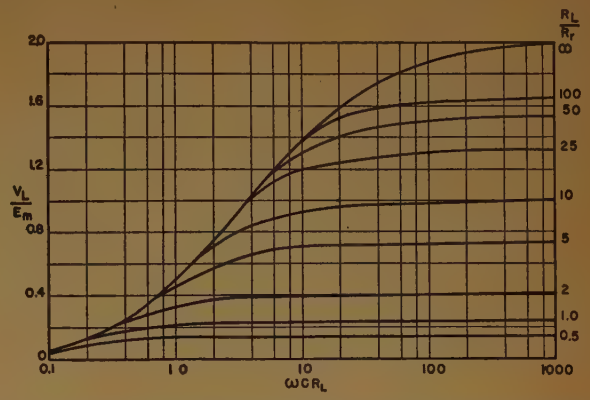

Fig. 8—The load-voltage ratio curves of the full-wave doubler.

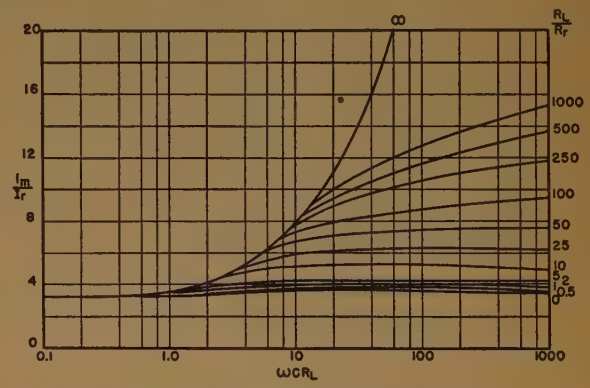

Fig. 9—The maximum tube-current ratio curves of the full-wave doubler.

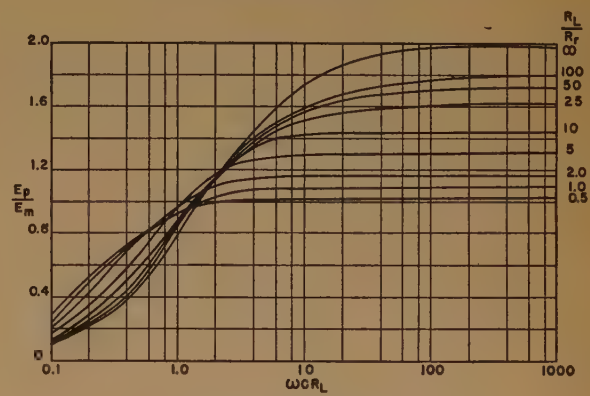

Fig. 10—The inverse peak-voltage ratio curves of the full-wave doubler.

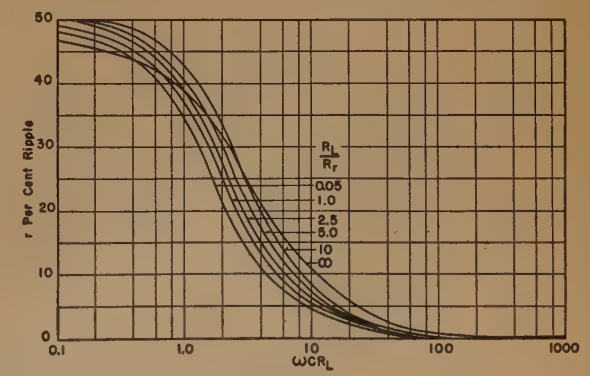

Fig. 11—The per cent ripple curves of the full-wave doubler.

circuits, and other means must be used. Other analyses[3,4] have shown that the ripple is 121.2 per cent for the half-wave and 48.3 per cent for the full-wave circuits at ωCR_L equal to zero, and this is apparently a general characteristic of the circuits so long as the filter is pure capacitance.

The condition that the voltage across the condensers is constant at ωCR_L equal to infinity serves in all cases, and this end point is of more importance than the other. For, if these infinite limit end points are determined, the characteristics for V_L/E_m and E_p/E_m, in most practical cases, are determined because the characteristics of the circuits for all except the higher values

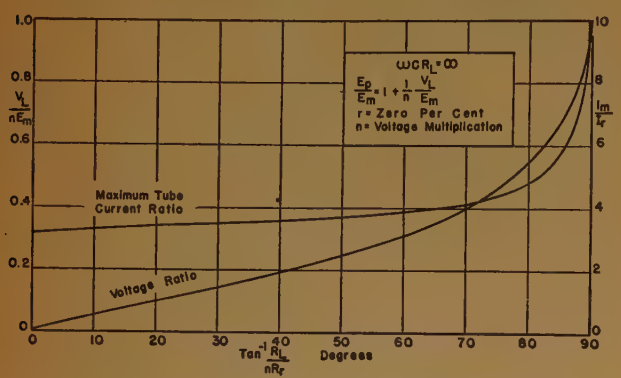

Fig. 12—The limiting characteristics of rectifiers with a voltage multiplication of n times.

of R_L/R_r are almost flat from the region of $\omega CR_L=\infty$ down through the operating range. The i_m/I_r characteristics have more slope, and consequently the determination of the end point may not give a value sufficiently near to the actual value of the characteristic; however, if only a knowledge of the region of operation is required, the value of the end point at $\omega CR_L=\infty$ will almost always serve.

The General Voltage-Multiplying Circuit

The investigation of the voltage ratio characteristic end points at $\omega CR_L=\infty$ yields the same form of equations for all half-wave voltage multiplier circuits. The equations differ only by a constant of multiplication in each case. The general form is

$$(1/n)(R_L/R_r)=F(V_L/nE_m)$$

where

$$F(V_L/nEm)=\frac{2\pi(V_L/nE_m)}{2\sqrt{1-(V_L/nE_m)^2}-(V_L/nE_m)[\pi-2\sin^{-1}(V_L/nE_m)]}$$

and n is the multiplication factor or number of tubes. The inverse peak-voltage-ratio end points, in each case, then become $E_p/E_m=1+(V_L/nE_m)$, and the maximum tube-current-ratio end points may be found from $i_m/I_r=(1/n)(R_L/R_r)[(nE_m/V_L)-1]$. The ripple in all cases is zero per cent.

These relations allow the development of curves for end points at $\omega CR_L=\infty$ which apply to half-wave voltage-multiplying circuits in general, and these curves are shown in Fig. 12.

Of all the single-phase rectifier circuits capable of voltage multiplication by a factor greater than 1, the half-wave circuits are the most common. The half-wave doubler, is the most elementary of the family and is shown in Fig. 13(a). It is distinctive of the half-wave multipliers that the addition of a stage consisting of one diode and one condenser to an existing circuit will increase the multiplication of the circuit by a factor of 1. A family of such circuits may be constructed. Fig. 13(b) shows a series of such stages so that if the diode resistance is low and the load resistance high, a steady voltage of approximately $6E_m$ may be obtained between points P_6 and G. Even voltage multiples are obtained between P_n and G, n being even; odd multiples are obtained between P_n and X, n being odd. These voltages are a steady direct voltage; however, an oscillating voltage with a peak value of $(n+1)E_m$ when n is even may be obtained between P_n and X.

Fig. 12 shows end points for circuits of this type. Values of ωCR_L used are usually so large that the operation points of such circuits may be determined from the end-point curves alone. This is important because the circuits are such that a complete analysis would be very difficult if not impossible. Such circuits have been mentioned by several authors[6-9] but no attempts have been

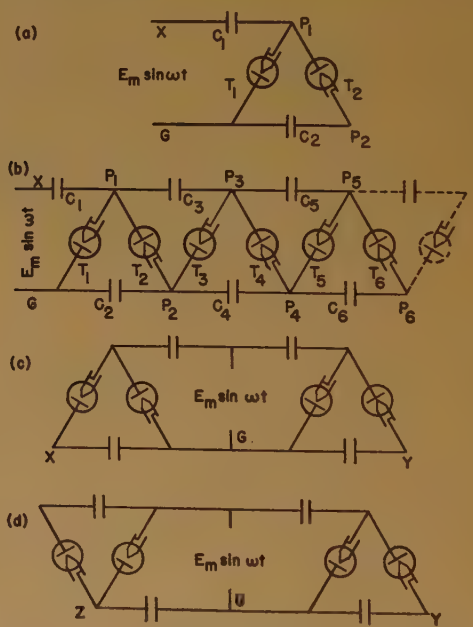

Fig. 13—Various voltage-multiplying rectifier circuits.
(a) half-wave doubler
(b) half-wave multiplier
(c) full-wave multiplier (quadrupler)
(d) hybrid multiplier (tripler)

[6] H. Greinacher, "Über eine Methode, Wechselstrom mittels elektrischer Ventile und Kondensatoren in hochgespannten Gleichstrom umzuwandeln," *Zeit. für Phys.*, vol. 4, pp. 195–205; February, 1921.

[7] J. D. Cockcroft and E. T. S. Walton, "Experiments with high velocity positive ions—Further developments in the method of obtaining high velocity positive ions," *Proc. Royal Soc.*, series A, vol. 136 pp. 619–630; June, 1932.

made to advance a method of predetermining the operation of a given circuit of this type beyond the doubler or tripler stages.

Voltage-multiplication circuits may be designed to operate in such a manner that they may be classed as full-wave multipliers. One such circuit, the full-wave doubler, is shown in Fig. 1(b). Another full-wave multiplier is illustrated in Fig. 13(c) and is a voltage quadrupler, i.e.: a voltage of approximately $4E_m$ may be obtained between X and Y if the R_L/R_r ratio and ωCR_L are large.

Examination of the circuit of Fig. 13(c) shows that the section to the right of the alternating-current input terminals is the half-wave doubler of Fig. 13(a). The section of the circuit to the left of the input is similar with the exception that the tubes are reversed in position. In operation, the circuit may be compared with two half-wave doublers arranged so that one of the output condensers receives charge every half cycle and the charges are equal in magnitude. Then if point Y is at a potential of approximately $2E_m$ positive with respect to G, point X is at a similar potential negative with respect to G. In the steady state, a potential of approximately $4E_m$ obtains between X and Y. The circuit may be expanded as previously described for half-wave circuits. In such a case, two stages, one on each side of the alternating-current input, are added simultaneously, and the load is connected between the extreme terminals of the circuit. It is possible to have a different number of stages on each side of the alternating-current input, but an unbalance in the output voltage would result with a fundamental component appearing in the output ripple voltage.

The circuit of Fig. 13(c) will give only even-integral voltage multiplications. Odd-integral multiplications may be obtained by means of the circuit of Fig. 13(d), a tripler. As in the previous case, the section to the right of the input is the half-wave doubler, while the section to the left appears as the same circuit inverted and with the tubes reversed. Although this arrangement is such that one of the load condensers receives charge every half cycle, it cannot be called a full-wave circuit because the charges are not of equal magnitude.

Although the curves of Fig. 12 were developed for half-wave multipliers, they can be extended to full-wave circuits because the operation of the full-wave circuits is similar to two half-wave circuits in series.

If V_L' is the voltage between the terminals of the full-wave multiplier, then $V_L = (1/2)\,V_L'$ is the voltage supplied by one of the half-wave multiplier sections. Each of the two half-wave sections loses an equal amount of charge to the load resistance. Therefore, the half-wave section operates as though it worked into a load resistance $R_L = (1/2)R_L'$ where R_L' is the complete load resistance. If the complete multiplication of the full-wave circuit is n', the multiplication of one of the half-wave sections is $n = (1/2)n'$.

By the same reasoning, the end point of the voltage-ratio characteristic for ωCR_L approaching infinity for one of the half-wave sections is given in Fig. 12 by the co-ordinates $\tan^{-1}(R_L/nR_r)$, (V_L/nE_m). If these co-ordinates are evaluated in terms of the full-wave multiplier, they become $\tan^{-1}(R_L'/n'R_r)$, $(V_L'/n'E_m)$. Then the voltage ratio curve of Fig. 12 must be applicable to full-wave multiplier circuits as well as half-wave circuits. This same reasoning may be applied to the circuits typified by Fig. 13(d).

The above process may be carried through to show that the curves of Fig. 12 apply to the full-wave circuits of Fig. 13 for the inverse peak voltage and maximum tube-current ratios. It must be borne in mind, however, that the slope of the current ratio characteristic is considerable in comparison with the other characteristics, and, therefore, the end-point calculation, singly, does not give a good approximation to the value of the characteristic unless the operating point is very high on the $\tan^{-1}(R_L/R_r)$ scale. However, the full-wave-circuit current characteristics have less slope in the operating range than do the half-wave circuits.

Appendix A—Nomenclature

C = capacitance of either condenser in half-wave and full-wave doubler circuits.
C_1 = capacitance of input condenser in half-wave doubler circuit; also one of the output condensers in the full-wave doubler circuits.
C_2 = capacitance of output condenser in half-wave doubler circuit and of the other condenser in the full-wave doubler circuit.
E_m = the maximum value of the alternating supply voltage.
E_p = the inverse peak voltage across a diode.
e_L = the instantaneous voltage across the load resistance.
e_p = the instantaneous inverse voltage across a diode.
I = the average direct load current.
I_e = the effective load current.
I_r = the average tube current.
i = the instantaneous load current.
i_m = the maximum tube current.
R_L = the resistance of the load.
R_r = the resistance of a diode.
r = per cent ripple in load voltage.
t = time in seconds.
V_L = the average direct voltage across the load.
V_1 = the voltage across condenser C_1.
V_2 = the voltage across condenser C_2.
α = the angle in radians at which a diode begins to conduct current.
β = the angle in radians at which a diode ceases to conduct current.
$\omega = 2\pi f$, where f is the frequency of the alternating-current supply in cycles per second.

[8] S. Gradstein, "Modern high-voltage equipment," *Philips Tech. Rev.*, vol. 1, pp. 6–10; January, 1936.
[9] D. L. Waidelich, "Voltage multiplier circuits," *Electronics*, vol. 14, pp. 28–29; May, 1941.

Appendix B—Analysis

The case of the half-wave voltage doubler will be treated in detail. All other circuits were solved by the same methods. In the circuit of Fig. 1(a) let V_1, positive on the right, be the voltage across the input condenser and V_2, positive at the top, be the voltage across the load condenser. Let R_r be in series with each cathode. Let i_1 flow in the input loop, positive in the plate to cathode direction, and i_2 flow in the loop formed by the load and the two tubes, also positive in plate-to-cathode direction.

A. Ratio of the average direct voltage to the maximum of the alternating input voltage.

1. $\omega CR_L = \infty$.

The condensers are assumed to be so large that a constant voltage V_1 is maintained across the input condenser and a constant voltage V_2 across the load.

By Kirchhoff's equations,

$$E_m \sin \omega t - i_1 R_r - V_1 = 0 \tag{1}$$

and

$$E_m \sin \omega t + V_2 + i_2 R_r - V_1 = 0. \tag{2}$$

The tube T_1 starts to conduct at the angle

$$\omega t = \alpha_1 = \sin^{-1} (V_1/E_m)$$

and stops conducting at the angle $\omega t = \beta_1 = \pi - \alpha_1$. The average current is then

$$I_1 = \frac{1}{2\pi R_r} \int_{\alpha_1}^{\beta_1} (E_m \sin \omega t - V_1) d(\omega t)$$

or

$$I_1 = \frac{1}{2\pi R_r} \left\{ 2\sqrt{E_m^2 - V_1^2} - V_1 [\pi - 2 \sin^{-1} (V_1/E_m)] \right\}. \tag{3}$$

The tube T_2 starts conducting at

$$\omega t = \alpha_2 = \sin^{-1} (V_1 - V_2)/E_m$$

and ends at $\omega t = \beta_2 = \pi - \alpha_2$. The average current is

$$I_2 = \frac{1}{2\pi R_r} \left\{ 2\sqrt{E_m^2 - (V_1 - V_2)^2} - (V_2 - V_1) \left[\pi - 2 \sin^{-1} \left(\frac{V_2 - V_1}{E_m} \right) \right] \right\}. \tag{4}$$

The average current must also satisfy

$$I_1 = I_2 = V_2/R_L. \tag{5}$$

Equating (3) and (4) to (5) and solving the equations obtained,

$$V_2/E_m = V_L/E_m = 2V_1/E_m \tag{6}$$

and

$$\left(\frac{1}{2\pi}\right)\left(\frac{R_L}{R_r}\right) = \frac{2(V_1/E_m)}{2\sqrt{1 - (V_1/E_m)^2} - (V_1/E_m)[\pi - 2 \sin^{-1} (V_1/E_m)]}. \tag{7}$$

Equations (6) and (7) relate the parameter (R_L/R_r) with the ratio of the average direct voltage across the load to the maximum of the alternating input voltage. This relationship is plotted in Fig. 4.

2. $\omega CR_L = 0$.

Assuming zero capacitance, the input is open-circuited and $V_L/E_m = 0$ per cent for all values of (R_L/R_r).

B. Ratio of the maximum tube current to the average tube current.

1. $\omega CR_L = \infty$.

Substituting (6) into (1) and (2), it is found that the same maximum current appears in each tube

$$i_m = (1/R_r)(E_m - V_1) \tag{8}$$

then

$$i_m/I_r = (1/2)(R_L/R_r)[(E_m/V_1) - 1]. \tag{9}$$

Equation (9) becomes indeterminate at (V_1/E_m) equal to zero, and the limit of (i_m/I_r) as (V_1/E_m) approaches zero is found from the conditionth at, as V_1 approaches zero, R_L approaches zero and

$$I_r = \frac{1}{2\pi} \int_0^{\pi} \frac{E_m}{R_r} \sin \omega t d(\omega t)$$

which integrates to $I_r = (1/\pi)(E_m/R_r)$;

therefore,

$$i_m/I_r = \pi. \tag{10}$$

2. $\omega CR_L = 0$.

The circuit becomes nonoperative when the capacitance is made zero, and the limit of the current ratio as ωCR_L approaches zero is determined in the same manner as given in the solution for the lower limit of the $\omega CR_L = \infty$ curve

$$i_m/I_r = \pi. \tag{11}$$

C. Ratio of the inverse peak voltage to the maximum of the alternating input voltage.

1. $\omega CR_L = \infty$.

The instantaneous inverse voltage across the diode is $e_p = E_m \sin \omega t + (V_L/2)$, and the peak inverse voltage is then $E_p = E_m + (V_L/2)$.

Thus,

$$E_p/E_m = 1 + (V_L/2E_m) \tag{12}$$

where relative values of (R_L/R_r) and (V_L/E_m) are determined by (6) and (7).

2. $\omega CR_L = 0$.

The circuit becomes nonoperative and $E_p/E_m = 0$.

D. Per cent ripple in load voltage.

1. $\omega CR_L = \infty$.

Since an infinite capacitance is assumed, $r = 0$ per cent.

2. $\omega CR_L = 0$.

The limit of the ripple percentage as ωCR_L approaches zero is determined by making C_2, the capacitance across the load, zero and allowing C_1 to remain finite. The equivalent circuit for conduction of current through R_L then consists of C_1, R_r, and R_L in series and

$$i = (E_m/z) \sin (\omega t + \theta)$$

with

$$I_e^2 = \frac{1}{2\pi} \left(\frac{E_m}{z} \right)^2 \int_{-\theta}^{\pi - \theta} \sin^2 (\omega t + \theta) d(\omega t)$$

which gives

$$I_e^2 = (1/4)(E_m/z)^2. \tag{13}$$

The average current is

$$I = \frac{1}{2\pi} \left(\frac{E_m}{z} \right) \int_{-\theta}^{\pi - \theta} \sin (\omega t + \theta) d(\omega t)$$

which integrates to

$$I = (1/\pi)(E_m/z). \tag{14}$$

By the use of (13) and (14) substituted into

$$r = (100/I)\sqrt{I_e^2 - I^2}$$

the ripple is $r = 121.2$ per cent. The ripple is then independent of C_1.

Current and Power in Velocity-Modulation Tubes*

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Summary—**This paper gives a physical and mathematical analysis of velocity modulation. The velocity modulation is expressed in terms of a variation with time of density at any distance from the modulating source. This time-density variation is developed as a Fourier series having Bessel coefficients and has the same form as Webster's expression for current. The high-frequency current and power induced in a resonant circuit by the variable-density electron stream are calculated. The effects of transit time through the resonant circuit of the modulating source and of the output circuit are included in the calculations. The possibility of using a steady stream of electrons passing through a single resonator as a transit-time oscillator is discussed.**

Introduction

THE physical concepts which underlie the operation of velocity-modulation devices such as the Klystron are relatively new[1,2] and quite different from those applicable to the usual vacuum tube, and the already extensive literature on the subject does not seem to contain an adequate treatment of these concepts. Webster's mathematical analysis of the Klystron,[3] which has been commonly followed,[4] gives correct results under certain limiting conditions, but the derivation is open to certain misinterpretations.[5] Previous treatments also have neglected the effects of transit time between the grids which produce the velocity modulation and between those which utilize it, and the results, for some practical applications, are therefore only approximations. The present paper is intended to give a fairly complete physical and mathematical treatment of the mechanism of velocity modulation and an analysis of the process in which power is delivered to a tuned circuit by the modulated electron stream.

In a velocity-modulation tube an electron stream is projected through two sets of grids. The first set, referred to as the buncher, produces the velocity modulation, while the second set, or catcher, receives high-frequency energy from the modulated stream. As illustrated in Fig. 1, the electrons all enter the buncher with the velocity v_0, determined by the direct accelerating voltage E_0, but they emerge with varying velocities because of the alternating voltage applied between the two grids. The variation of velocity produces bunching of the electrons. The bunching produced at any point, distant s from the buncher, can be expressed as a variation of electron density with time at this point. The variable-density stream, passing through the two grids

Fig. 1—Schematic diagram of a velocity-modulation tube.

of the catcher, induces a current in the catcher circuit. The first problem is to find the variation of electron density as a function of time, and the second problem is to find the current induced in the catcher by the variable-density electron stream.

Modulation of the Electron Stream

The velocity of the electrons emerging from the buncher at time t_1 is

$$v = \sqrt{(2q/m)E_0[1 + (E_1/E_0)\sin \omega t_1]} = v_0\sqrt{1 + (E_1/E_0)\sin \omega t_1} \quad (1)$$

where q and m are the charge and mass of an electron, and

$$v_0 = 5.93 \cdot 10^7 \sqrt{E_0 \text{ volts}} \text{ centimeters per second.} \quad (2)$$

The subscript (1) will be used to designate quantities measured at the buncher $x=0$, while the subscript (2) will be reserved for quantities measured at the catcher, $x=s$. For $E_1/E_0 \ll 1$ this becomes

$$v = v_0(1 + (1/2)(E_1/E_0)\sin \omega t_1) = v_0(1 + (v_1/v_0)\sin \omega t_1) \quad (3)$$

where

$$v_1/v_0 = (1/2)(E_1/E_0) \quad (4)$$

and is small compared with unity.

The charge density at $x=s$ is

$$\rho_2 = \operatorname{limit}_{\Delta x_2 \to 0} \frac{qN_2}{\Delta x_2} \quad (5)$$

where N_2 is the number of electrons in a volume element of length Δx_2 and cross section equal to that of the beam. Since the velocity modulation v_1/v_0 is assumed small

$$\operatorname{limit}_{\Delta x_2 \to 0} \frac{qN_2}{\Delta x_2} = \operatorname{limit}_{\Delta t_2 \to 0} \frac{qN_2}{v_0 \Delta t_2} \quad (6)$$

where Δt_2 is the interval of time required for the group of N_2 electrons to pass the plane at $x=s$.

The time t_2 at which an electron reaches s is equal to the time t_1 at which it emerged from the buncher, plus the transit time:

$$t_2 = t_1 + \frac{s}{v_0(1 + (v_1/v_0)\sin \omega t_1)} \approx t_1 + s/v_0 - (sv_1/v_0^2)\sin \omega t_1. \quad (7)$$

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[1] W. C. Hahn and G. F. Metcalf, "Velocity-modulated tubes," PROC. I.R.E., vol. 27, pp. 106–116; February, 1939.

[2] R. H. Varian and S. H. Varian, "High-frequency oscillator and amplifier," *Jour. Appl. Phys.*, vol. 10, pp. 321–327; May, 1939.

[3] D. L. Webster, "Cathode ray bunching," *Jour. Appl. Phys.*, vol. 10, pp. 501–508; July, 1939.

[4] R. I. Sarbacher and W. A. Edson, "Tubes employing velocity modulation," PROC. I.R.E., vol. 31, pp. 439–452; August, 1943.

[5] J. G. Brainerd, Glenn Koehler, Herbert J. Reich, and L. F. Woodruff, "Ultra-high-frequency techniques," D. Van Nostrand Co., New York, N. Y., 1942, p. 334.

An electron arriving at $x=s$ during the interval Δt_2 may have emerged from the buncher at any time t_1 which will satisfy (7). Since electrons leave the buncher at a uniform rate, the number of electrons comprising N_2 may readily be computed after the interval or intervals during which they left the buncher are identified. The intervals in question are shown in Fig. 2(C).

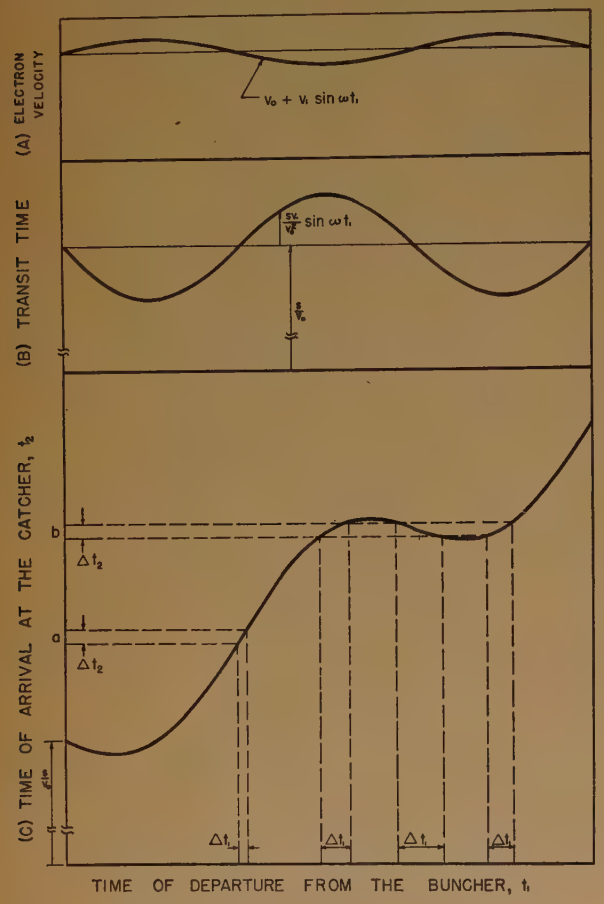

Fig. 2—Fundamental relations of velocity modulation.

In Fig. 2(A) are plotted the velocities of the electrons against the time of departure from the buncher. The number of electrons leaving the buncher per second is constant and equal to i_0/q where i_0 is the beam current. The variation of the velocity with departure time, t_1, results in a variation of the "transit time" required to travel the distance s. This transit time is plotted as a function of departure time in Fig. 2(B).

Fig. 2(C) is a graph of (7) and shows the times of arrival, at $x=s$, of the different electrons as a function of their times of departure from $x=0$. The curve is plotted for a particular value of buncher voltage E_1 sufficiently large so that certain accelerated electrons actually overtake and pass other electrons which left the buncher at earlier times. Thus the electrons which arrive during the interval Δt_2 at time a left the buncher during the corresponding single interval Δt_1. However, those which arrive during the interval Δt_2 at time b left the buncher during three separate intervals, as shown by the dotted lines.

The number N_2 which appears in (6) is

$$N_2 = \sum \frac{i_0}{q} \Delta t_1 = \frac{i_0}{q} \sum \Delta t_1 \qquad (8)$$

where the summation includes all intervals Δt_1 corresponding to the particular interval Δt_2 in question. Upon substitution, the charge density at $x=s$ becomes[6]

$$\rho_2 = \operatorname*{limit}_{\Delta t_1 \to 0} \frac{i_0}{v_0} \frac{\sum \Delta t_1}{\Delta t_2} = \frac{i_0}{v_0} \sum \left(\operatorname*{limit}_{\Delta t_1 \to 0} \frac{\Delta t_1}{\Delta t_2} \right). \qquad (9)$$

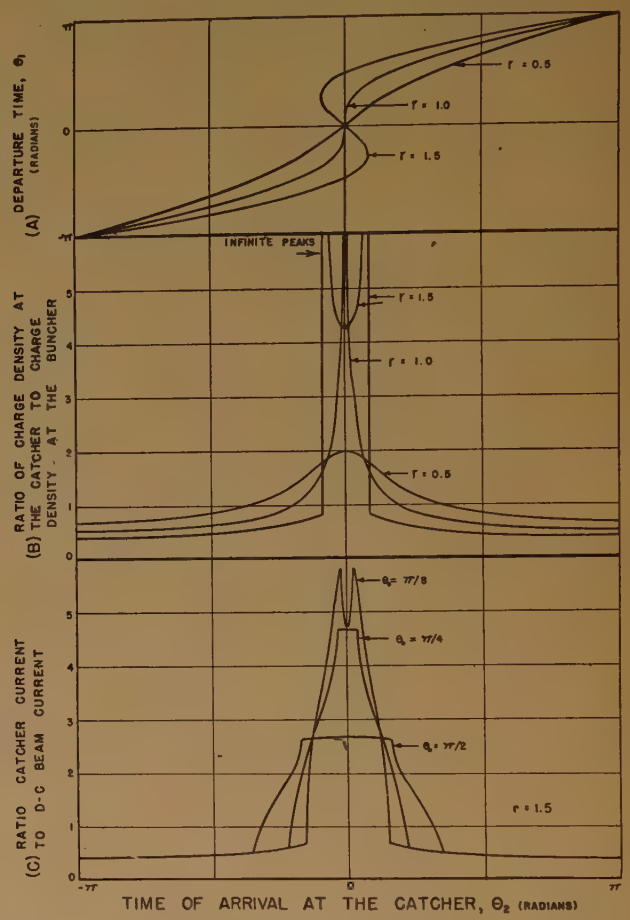
(A) DEPARTURE TIME, θ_1 (RADIANS)
r = 0.5
r = 1.0
r = 1.5
INFINITE PEAKS
(B) RATIO OF CHARGE DENSITY AT THE CATCHER TO CHARGE DENSITY AT THE BUNCHER
r = 1.5
r = 1.0
r = 0.5
(C) RATIO CATCHER CURRENT TO D-C BEAM CURRENT
$\theta_1 = \pi/8$
$\theta_1 = \pi/4$
$\theta_1 = \pi/2$
r = 1.5
TIME OF ARRIVAL AT THE CATCHER, Θ_2 (RADIANS)

Fig. 3—Variation of charge density with time at the catcher and wave forms of the catcher current.

The ratios $\Delta t_1/\Delta t_2$ are to be computed from (7). If both sides of (7) are multiplied by ω, and a new variable introduced it becomes

$$\theta_2 = \theta_1 - r \sin \theta_1 \qquad (10)$$

where $\theta_1=\omega t_1$, $\theta_2=\omega(t_2-s/v_0)$ and $r=\omega s v_1/v_0^2$. Equation (9) now becomes

$$\rho_2 = \frac{i_0}{v_0} \sum \operatorname*{limit}_{\Delta\theta_1 \to 0} \frac{\Delta\theta_1}{\Delta\theta_2}. \qquad (11)$$

If θ_1, in (10), is plotted against θ_2 the graph of Fig. 3(A) results, and the ratio $\Delta\theta_1/\Delta\theta_2$ becomes, in the limit, the slope of the curves. Curves are plotted for three different

[6] Webster (see footnote reference 3) writes an equation similar to (9) but without the summation sign, stating that it follows from continuity. Such an equation can be derived from the conservation of charge only upon the assumption that the velocity modulation is sufficiently small so that electrons do not overtake and pass others which left the buncher at earlier times.

values of r, 0.5, 1.0, and 1.5. It will be noted that for $r<1$ the slope of the curve is always positive and finite. For $r=1$ the slope becomes infinite at one point in the cycle. This implies an infinite charge density, which means simply that electrons which left the buncher one after the other arrive simultaneously. Infinite charge density can, of course, never actually exist because of the finite size of the electron. For $r>1$ the slope of the curve is negative at certain points. This is not to be interpreted as a negative charge density. It means that electrons which left the buncher in one sequence arrive in the reverse order. For this reason, the magnitude, only, of the slope must be used in computing the charge density from (11). Thus, (11) should be written

$$\rho_2 = \frac{i_0}{v_0} \sum \operatorname*{limit}_{\Delta\theta_1 \to 0} \left| \frac{\Delta\theta_1}{\Delta\theta_2} \right| . \tag{12}$$

A plot of the charge density against θ_2 for the three values of r is shown in Fig. 3(B). Since θ_2 is proportional to t_2 this graph shows how the density of charge arriving at $x=s$ varies with time. The wave form of the catcher current, which is studied in a later section, is plotted in Fig. 3(C).

From (10)

$$\operatorname*{limit}_{\Delta\theta_1 \to 0} \left| \frac{\Delta\theta_1}{\Delta\theta_2} \right| = \left| \frac{1}{1 - r\cos\theta_1} \right| . \tag{13}$$

Therefore the charge density as a function of θ_2 can be expressed by the parametric equations

$$\rho_2 = \frac{i_0}{v_0} \sum \left| \frac{1}{1 - r\cos\theta_1} \right|$$
$$\theta_2 = \theta_1 - r\sin\theta_1. \tag{14}$$

For a particular value of θ_2 there may be several values of θ_1. Each such value contributes a term to the summation in the expression for ρ_2.

The charge density ρ_2 can be expressed explicitly in terms of θ_2 by expanding it in a Fourier series:

$$\rho_2 = a_0 + a_1\cos\theta_2 + \cdots + a_n\cos n\theta_2 + \cdots + b_1\sin\theta_2 + \cdots + b_n\sin n\theta_2 + \cdots$$

where

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} \rho_2 \cdot |\, d\theta_2| = \frac{1}{2\pi}\frac{i_0}{v_0}\int_0^{2\pi} \left| \frac{1 - r\cos\theta_1}{1 - r\cos\theta_1} \right| d\theta_1 = \frac{i_0}{v_0} .$$

The summation of (14) is automatically taken care of when the variable of integration is changed since all values of θ_1 will be included in the integration.

$$a_n = \frac{1}{\pi}\int_0^{2\pi} \rho_2 \cdot \cos n\theta_2 \cdot d\theta_2$$
$$= \frac{1}{\pi}\frac{i_0}{v_0}\int_0^{2\pi} \cos n(\theta_1 - r\sin\theta_1)\cdot d\theta_1$$
$$= 2\frac{i_0}{v_0} J_n(nr)$$

where $J_n(nr)$ is the Bessel function of the first kind of order n.

$$b_n = \frac{1}{\pi}\int_0^{2\pi} \rho_2 \cdot \sin n\theta_2 \cdot d\theta_2$$
$$= \frac{1}{\pi}\frac{i_0}{v_0}\int_0^{2} \sin n(\theta_1 - r\sin\theta_1)\cdot d\theta_1 = 0.$$

Therefore,

$$\rho_2 = \frac{i_0}{v_0}\left[1 + 2\sum_{n=1}^{\infty} J_n(nr)\cos n\theta_2 \right]. \tag{15}$$

In this equation $\theta_2 = \omega t_2$ and

$$r = \omega s v_1/v_0^2 = v_1/v_0(\omega s/v_0) = (1/2)(E_1/E_0)(\omega s/v_0).$$

The current induced in a tuned circuit by this variable-density stream of electrons is calculated in the next section, following a method applied by North[7] to the general problem of transit-time phenomena between parallel-plane electrodes.

The Catcher Current

In order to abstract energy from the electron stream it is passed through two grids connected to a tuned circuit as shown in Fig. 4. As a small quantity of electricity Δq passes from $x=0$ to $x=d$, a current Δi, is

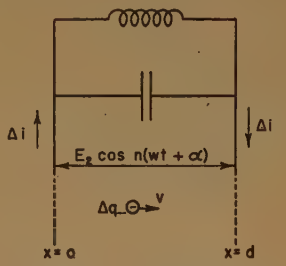

Fig. 4—The catcher circuit.

induced in the pair of grids.[8] This element of current is

$$\Delta i = (\Delta q)v/d = (\rho v_0 \Delta t)v/d. \tag{16}$$

Since the velocity modulation produced by the buncher is usually small, the velocity of all electrons entering the first grid has been taken as equal to the average velocity v_0. This approximation is equivalent to neglecting second-order terms in the final result. The total current at any time t is obtained by integrating the effects of all the charges moving in the space between the grids. These charges have entered the space during the interval τ beginning at time $t-\tau$, hence the current is

$$i = \frac{v_0}{d}\int_{t-\tau}^{t} \rho_{t_0} v_t dt_0 \tag{17}$$

where t_0 is the time a particular element of charge entered the space, and v_t is its velocity at the time t, for which the current is computed. ρ_{t_0} is the charge density at $x=0$ at the time t_0. The interval τ is the transit time of the electrons which are just leaving the space through the second grid at the time t. Due to the acceleration produced by the alternating voltage between the grids,

[7] D. O. North, "Analysis of the effects of space charge on grid impedance," Proc. I.R.E., vol. 24, pp. 108–136; January, 1936.

[8] C. K. Jen, "On the induced current and energy balance in electronics," Proc. I.R.E., vol. 29, pp. 345–349; June, 1941. See also the references given.

the velocity of a particular electron, and the transit times of different electrons, vary with time. The acceleration produced by the alternating voltage is

$$d^2x/dt^2 = [(-q)E_2/md]\cos n(\omega t + \alpha) \qquad (18)$$

where $-q$ is the charge on the electron, and $E_2 \cos n(\omega t+\alpha)$ is the voltage of the first grid with respect to the second grid. Let $E_2=P_2E_0$, where $P_2 \leqq 1$. Then since $2qE_0/m=v_0^2$, (18) becomes

$$\begin{aligned} d^2x/dt^2 &= -(v_0^2P_2/2d)\cos n(\omega t + \alpha) \\ &= -(v_0P_2/2\tau_0)\cos n(\omega t + \alpha) \end{aligned} \qquad (19)$$

where $\tau_0=d/v_0$ and will be referred to as the normal transit time. Integrating (19) the velocity is

$$v_t = dx/dt = -(v_0P_2/2\tau_0 n\omega)[\sin n(\omega t + \alpha) - \sin n(\omega t_0 + \alpha)] + v_0. \qquad (20)$$

Integrating again, the position, x, measured from the first grid, is

$$x = (v_0P_2/2\tau_0(n\omega)^2)[\cos n(\omega t + \alpha) - \cos n(\omega t_0 + \alpha) + n\omega(t - t_0)\sin n(\omega t_0 + \alpha)] + (t - t_0)v_0. \qquad (21)$$

From (15) the charge density passing $x=0$ at time t_0 is

$$\rho_{t_0} = \frac{i_0}{v_0}\left[1 + 2\sum_{n=1}^{\infty} J_n(nr)\cos n\omega t_0\right]. \qquad (22)$$

From (17) the current is

$$i = \frac{v_0}{d}\int_{t-\tau}^{t}\frac{i_0}{v_0}\left[1+2\sum_{n=1}^{\infty}J_n(nr)\cos n\omega t_0\right]\left[-\frac{v_0P_2}{2\tau_0 n\omega}\{\sin n(\omega t+\alpha)-\sin n(\omega t_0+\alpha)\}+v_0\right]dt_0. \qquad (23)$$

Evaluation of the integral gives a direct current plus a series of alternating-current components. The particular component that delivers energy to the tuned circuit is the one having the frequency to which the circuit is tuned. Therefore, in evaluating the integral only terms contributing to this frequency are retained. These terms give a current i':

$$\begin{aligned} i' = i_0\{&\tau/\tau_0 + (2J_n(nr)/\tau_0 n\omega)[\sin n\omega t - \sin n\omega(t-\tau)] \\ &-(P_2/2(\tau_0 n\omega)^2)[\cos n(\omega t+\alpha) - \cos n(\omega t - \omega\tau+\alpha) \\ &+ \tau n\omega \sin n(\omega t+\alpha)]\}. \end{aligned} \qquad (24)$$

Since τ varies periodically the first term in the bracket contains alternating-current components. Letting $\omega\tau=\theta$ and $\omega\tau_0=\theta_0$, equation (24) can be expanded and written:

$$\begin{aligned} i' = i_0\Bigg[&\frac{\theta}{\theta_0} + \left\{2J_n(nr)\left[\cos\left(n\alpha+\frac{n\theta}{2}\right)\right]\left[\frac{\sin n\theta/2}{n\theta_0/2}\right] - \frac{P_2}{2(n\theta_0)^2}[1-\cos n\theta]\right\}\cos n(\omega t+\alpha) \\ &+\left\{2J_n(nr)\left[\sin\left(n\alpha+\frac{n\theta}{2}\right)\right]\left[\frac{\sin n\theta/2}{n\theta_0/2}\right] - \frac{P_2}{2(n\theta_0)^2}[n\theta-\sin n\theta]\right\}\sin n(\omega t+\alpha)\Bigg]. \end{aligned} \qquad (25)$$

The alternating-current components in the term θ/θ_0 can be obtained from (21), which becomes, for $x=d$,

$$1 = P_2/2(n\theta_0)^2[\cos n(\omega t+\alpha) - \cos n(\omega t+\alpha-\theta) + n\theta\sin n(\omega t+\alpha-\theta)] + \theta/\theta_0$$

or

$$\begin{aligned} \theta/\theta_0 = 1 &- (P_2/2(n\theta_0)^2)[1-\cos n\theta - n\theta\sin n\theta]\cos n(\omega t+\alpha) \\ &- (P_2/2(n\theta_0)^2)[n\theta\cos n\theta - \sin n\theta]\sin n(\omega t+\alpha). \end{aligned} \qquad (26)$$

Substituting (26) in (25) and omitting the direct-current term gives

$$\begin{aligned} i' = i_0&\left\{2J_n(nr)\left[\cos\left(n\alpha+\frac{n\theta}{2}\right)\right]\left[\frac{\sin n\theta/2}{n\theta_0/2}\right] - \frac{P_2}{2}\left[\frac{2(1-\cos n\theta)-n\theta\sin n\theta}{(n\theta_0)^2}\right]\right\}\cos n(\omega t+\alpha) \\ +i_0&\left\{2J_n(nr)\left[\sin\left(n\alpha+\frac{n\theta}{2}\right)\right]\left[\frac{\sin n\theta/2}{n\theta_0/2}\right] - \frac{P_2}{2}\left[\frac{n\theta(1-\cos n\theta)-2\sin n\theta}{(n\theta_0)^2}\right]\right\}\sin n(\omega t+\alpha). \end{aligned} \qquad (27)$$

Since the cosine term of (27) is in phase with the voltage $E_2 \cos n(\omega t+\alpha)$, the average power delivered to the circuit is

$$W = E_2 i_0\left\{J_n(nr)\left[\cos\left(n\alpha+\frac{n\theta_0}{2}\right)\right]\left[\frac{\sin n\theta_0/2}{n\theta_0/2}\right] - \frac{P_2}{4}\left[\frac{2(1-\cos n\theta_0)-n\theta_0\sin n\theta_0}{(n\theta_0)^2}\right]\right\} \qquad (28)$$

and the efficiency is

$$\eta = P_2\left\{J_n(nr)\left[\cos\left(n\alpha+\frac{n\theta_0}{2}\right)\right]\left[\frac{\sin n\theta_0/2}{n\theta_0/2}\right] - \frac{P_2}{4}\left[\frac{2(1-\cos n\theta_0)-n\theta_0\sin n\theta_0}{(n\theta_0)^2}\right]\right\} 100 \text{ per cent.} \qquad (29)$$

Here θ_0 has been written for θ. This substitution is equivalent to neglecting small components of current of frequencies different from $n\omega$ and hence has negligible effect upon the power or efficiency. The second term in the brackets of (28) and (29) is due to the additional modulation produced by the alternating voltage as the

electron stream passes through the catcher grids. For values of $n\theta_0$ less than 2π this term represents energy given to the electron stream by the tuned circuit, and hence a slight decrease in efficiency. The relatively small magnitude of this term is shown by the graph (for $P=1$) of Fig. 5. For certain values of $n\theta_0$ greater than

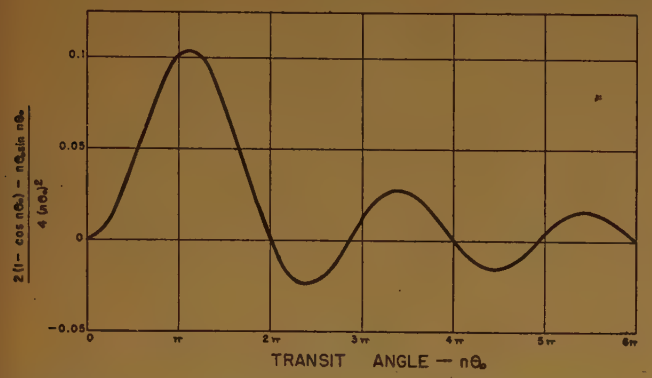

Fig. 5—Decrease in efficiency, due to modulation introduced by the catcher voltage for $E_2=E_0$, plotted against transit angle.

2π, this term shows energy to be abstracted from the electron stream, corresponding to the negative resistance encountered in diode analysis.[7,9,10]

Since the voltage is supported by energy abstracted from the electron stream, its phase angle α will be determined by the condition $\cos(n\alpha+n\theta_0/2)=1$. For this condition, (29) is

$$\eta = P_2\left\{J_n(nr)\frac{\sin n\theta_0/2}{n\theta_0/2} - \frac{P_2}{4}\left[\frac{2(1-\cos n\theta_0)-n\theta_0\sin n\theta_0}{(n\theta_0)^2}\right]\right\} 100 \text{ per cent.} \quad (30)$$

The efficiency will be a maximum for the n^{th} harmonic when $J_n(nr)$ is a maximum and $P_2=1$. The maximum values of $J_n(nr)$ and the corresponding values of r are given in Table I. The maximum efficiencies for operation at the fundamental or at various harmonics are plotted as functions of the transit angle in Fig. 6. For zero transit angle the catcher current is simply the charge density times the velocity v_0. This assumption is implicit in the analyses of Webster[8] and of Condon[11] and the efficiencies they give agree with the initial points on the curves of Fig. 6. In a practical design, the transit angle may be $\pi/2$ or larger. A transit angle of $\pi/2$ would correspond to a spacing of approximately 1.5 millimeters in a 10-centimeter Klystron operating at a plate potential of 1000 volts. If this transit angle of $\pi/2$ is neglected in calculating the maximum efficiency for the fundamental, the value obtained will be about 21 per cent high.[12]

TABLE I

n	1	2	3	4	5	10
Max. $J_n(nr)$	0.58	0.49	0.43	0.40	0.37	0.30
r	1.84	1.52	1.40	1.33	1.28	1.20

Wave Form of Catcher Current

The wave form of the catcher current can be plotted by use of the curves of Fig. 3(A). If the interaction of the catcher voltage with the electron stream is neglected, (17) can be written

$$i = v_0\int_{t-\tau}^{t}\frac{\rho_t}{d}\frac{dx}{dt_0}dt_0 = v_0\int_0^d\frac{\rho_x}{d}dx. \quad (31)$$

In this expression, the integral is the average charge density in the catcher space. From (12) the average charge density over a time interval $\Delta\theta_2$ is

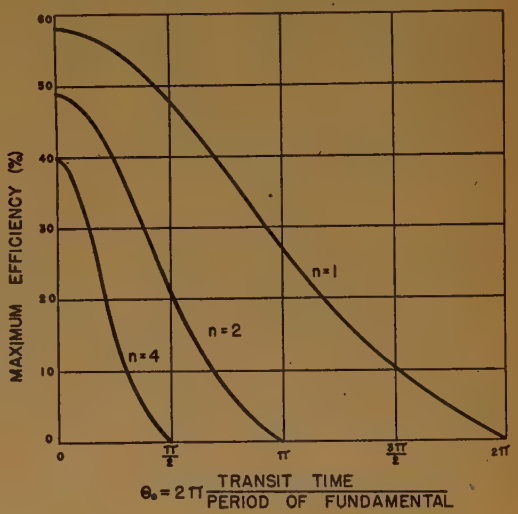

Fig. 6—Maximum efficiency as a function of transit angle.

$$\rho_2\ (\text{aver.}) = \frac{i_0}{v_0}\sum\left|\frac{\Delta\theta_1}{\Delta\theta_2}\right|. \quad (32)$$

The value of this average charge density is equal to the integral of (31) when $\Delta\theta_2$ is taken equal to the transit time between the catcher grids. Therefore, the catcher current can be written

$$i_2 = i_0\sum\left|\frac{\Delta\theta_1}{\Delta\theta_2}\right| \quad (33)$$

where the interval $\Delta\theta_2$ must be taken equal to θ_0.

Plots of the current wave form for transit angles of $\pi/8$, $\pi/4$, and $\pi/2$ are shown in Fig. 3(C). In a Klystron operating at 10 centimeters with a beam voltage of 1000 volts, these transit angles correspond to grid spacings of 0.375, 0.75, and 1.5 millimeters, respectively. It is apparent that in practical operation the wave form of the catcher current differs decidedly from that of the charge density; in particular, the infinite peaks present

[9] W. E. Benham, "Theory of the internal action of thermionic systems at moderately high frequencies," *Phil. Mag.*, vol. 5, pp. 641–662; March, 1928; and vol. 11, pp. 457–517; February, 1931.

[10] F. B. Llewellyn, "Operation of ultra-high-frequency vacuum tubes," *Bell Sys. Tech. Jour.*, vol. 14, pp. 632–665; October, 1935.

[11] E. U. Condon, "Electronic generation of electromagnetic oscillations," *Jour. Appl. Phys.*, vol. 11, pp. 502–506; July, 1940.

[12] H. E. Hollmann, "Theoretical and experimental investigations of electron motions in alternating fields with the aid of ballistic models," Proc. I.R.E., vol. 29, pp. 70–79; February, 1941. Hollmann gives, without derivation, an equation for efficiency which is intended to take account of transit angle, but which is apparently in error.

in the charge-density variation do not appear in the current.

Transit-Time Effects at the Buncher

Equations (1) and (3), which express the velocity of electrons emerging from the buncher, are based on the assumption of zero transit time between the two grids of the buncher. The effects of a finite transit time are to decrease the magnitude of the velocity modulation for a given modulating voltage, to shift the phase of the modulated velocity, and to introduce electronic loading of the driving source.

Let the modulating voltage be $E_1 \sin \omega t_1$ and be considered positive when it accelerates the electrons. The velocity of electrons emerging from the buncher grids can be obtained by substituting $\alpha = 90$ degrees in (20):

$$v_t = -(v_0 E_1/2\omega\tau_0 E_0)[\cos \omega t_1 - \cos \omega(t_1 - \tau_0)] + v_0 \quad (34)$$

where τ_0 is the transit time between grids, and $t_1 - \tau$ is written for t_0. Letting $\omega\tau_0 = \theta_0$, (34) can be written in the form

$$v_t = v_0[1 + (E_1/2E_0)\left(\frac{\sin\theta_0/2}{\theta_0/2}\right)\sin(\omega t_1 - (\theta_0/2))]. \quad (35)$$

For zero transit angle, (35) reduces to (3). A finite transit time through the buncher reduces the modulation by the same factor which, because of finite transit time through the catcher, appears in (27). The phase angle $\theta_0/2$ appearing in (35) can be taken into account by measuring the distance s from the center of the buncher, when this distance is used in over-all transit-time calculations.

The average power required from the source of buncher voltage can be obtained directly from the second term of (28), by substituting E_1 for E_2 and changing the sign:

$$W = E_0 i_0 \left\{\frac{E_1}{E_0}\left[\frac{2(1 - \cos\theta_0) - \theta_0 \sin\theta_0}{4\theta_0^2}\right]\right\} = \frac{E_1^2}{2} G \quad (36)$$

where E_1 is the peak value of the buncher voltage, and G is the conductance between the grids of the buncher due to the electronic loading. From (36), G can be written

$$G = \frac{i_0}{E_0}\left[\frac{2(1 - \cos\theta_0) - \theta_0 \sin\theta_0}{2\theta_0^2}\right]. \quad (37)$$

The magnitude of G in mhos for any transit angle θ_0 is equal to the corresponding ordinate of Fig. 5 multiplied by $2i_0/E_0$, since for this case $n = 1$. For a transit angle of $\pi/2$, corresponding to a grid spacing of 1.5 millimeters in a 10-centimeter Klystron operating at a potential, E_0, of 1000 volts, and a beam current of 50 milliamperes, this represents an electronic loading of 4.35 micromhos or 230,000 ohms. This value of resistance is of the same order of magnitude as the shunt impedance of the cavity resonators usually used for bunching.[13] For these operating conditions, the alternating-current power absorbed by the electron stream is but a very small percentage of the direct-current-power input $E_0 i_0$, amounting to 0.0435 per cent if $E_1 = E_0/10$.

It will be noted that the input resistance is negative for certain transit angles. This shows the possibility of using a steady stream of electrons passing through a single resonator as a transit-time oscillator. The criterion for sustained oscillations is that the positive shunt resistance of the resonator be not less than the negative shunt resistance introduced by the electron stream. For the typical operating conditions of $E_0 = 1000$ volts and $i_0 = 50$ milliamperes, the lowest negative resistance obtainable corresponds to a transit angle of $5\pi/2$, and has a value of 422,000 ohms. This value could be lowered by operating with a lower voltage and a higher beam current. From published values of the shunt resistance at resonance of cavity resonators,[13] it is not unreasonable to predict oscillations in such a system, although its efficiency would be low. A negative plate resistance is also obtained in a simple diode for the same transit angles which give a negative resistance between the buncher grids. This effect has been used to produce 10-centimeter oscillations in specially constructed diodes.[14] For the case of a temperature-limited diode, the expression for plate resistance would differ only by a constant from (37).[15]

[13] See page 331 of footnote reference 5.
[14] F. B. Llewellyn and A. E. Bowen, "The production of ultra-high-frequency oscillations by means of diodes," *Bell Sys. Tech. Jour.*, vol. 18, pp. 280–291; April 1939.
[15] See page 214 of footnote reference 7.

Correction

Paul E. Chamberlain has brought to the attention of the editors an error in "The Graphical Design of Cathode-Output Amplifiers," by David L. Shapiro, which appeared on pages 263 to 268 of the May, 1944, issue of the Proceedings. In column 2 on page 266, ninth line from the bottom, the sentence should read as follows:

"At point P, the handbook gives g_m as about 1600 micromhos. The plate current is 9 milliamperes and the screen current is 2.6 milliamperes, so the cathode current is 11.6 milliamperes, and the slope of the *cathode-current lines* will be $11.6/9 \times 1600 \times 10^{-6} = 2060 \times 10^{-6}$."

The correction is in italics.

A Mechanical Theory of Electron-Image Formation*

KURT SCHLESINGER†, ASSOCIATE, I.R.E.

Summary—**The formation of an electron image and its optical constants, such as magnification and position, is predicted from the simple assumption of transversal deflection in true proportion to the radial distance off the axis. The treatment is mechanical, rather than optical. Investigation of the electron trajectory is avoided.**

At first, the deflecting forces are supposed to act only in a negligible portion of the electron path. It is found that there exists for this case a "thin-lens law" which differs from the optical analogy only in that the distances are replaced by transit times. The resulting expression for electron-optical magnification is a ratio of transit times. Applications of these deductions are made for some particular electron tubes.

The mechanical treatment is then used in connection with an extended field with a variable deflection constant all along the axis. The constants of thick lenses, either electric, magnetic, or combined fields, are expressed in terms of three integrals of the deflection with respect to transit time. The first of these integrals gives the focal length, the second the magnification, and the third the positions of corresponding object and image planes. All results are available in a graph of a family of hyperbolas for various operation parameters. The plane accelerating field, which forms a virtual rather than a real electron image, is studied in the same manner.

Appendix A gives the mathematical proof of convergence of the series of functions used for approximation. Appendix B gives an analytical function for the potential distribution in the electron gun, which simplifies the discussion of such systems and helps to show that they act as positive lenses under all circumstances.

I. Introduction

MANY excellent discussions of the fundamentals of electron-optical instruments have been published in recent years.[1–8] The majority of these publications has emphasized analogies between electron optics and light optics. In dealing with electron lenses, the traditional method has been to determine the electron trajectories first, either theoretically by step-by-step methods,[4,5] or graphically by methods such as by trajectory plotters,[6,7] or, more recently, experimentally by the rubber model.[8] A few publications have discussed electron lenses without use of the electron path.[9,10]

* Decimal classification: R388×R583. Original manuscript received by the Institute, December 6, 1943; revised manuscript received, April 24, 1944. Presented, Indianapolis Section, Indianapolis, Ind., January 21, 1944. *Note:* The preparation of this paper was essentially completed prior to the author's connection with RCA Laboratories in October, 1941.

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[1] E. Bruche and O. Scherzer, "Elektronenoptik," Julius Springer, Berlin, Germany, 1934.

[2] I. G. Maloff and D. W. Epstein, "Electron Optics in Television," McGraw-Hill Book Company, New York, N. Y., 1938.

[3] J. Picht, "Elektronenoptik," Barth, Leipzig, Germany, 1939.

[4] R. Gans, "Electron paths in electron optics," *Zeit. für tech. Phys.*, vol. 18, pp. 41–48; 1937.

[5] O. Klemperer and W. D. Wright, "The investigation of electron lenses," *Proc. Phys. Soc.*, vol. 51, pp. 296–317; 1939.

[6] D. Gabor, "Mechanical tracer for electron trajectories," *Nature*, vol. 139, pp. 373–374; 1937.

[7] H. Salinger, "Tracing electron paths in electric fields," *Electronics*, vol. 10, pp. 50–54; October, 1937.

[8] P. H. J. A. Kleynen, "Tracing electron paths in electric fields," *Philips Tech. Rev.*, vol. D, pp. 321–352; 1937.

[9] K. Spangenberg and L. M. Field, "Some simplified methods of determining the optical characteristics of electron lenses," PROC. I.R.E., vol. 30, pp. 138–144; March, 1942.

[10] F. Gray, "Electrostatic electron optics," *Bell Sys. Tech. Jour.*, vol. 18, pp. 1–31; January, 1939

The present paper disregards all analogies with optics and treats the matter from a somewhat different point of view; the electron motion is being considered as a purely mechanical problem. This analysis yields a correlation between magnification and transit time, which proves useful in the design of electron-optical instruments, in particular those with thin lenses, even with combined electric and magnetic fields. This reasoning is then applied to paraxial rays in extended fields. It is possible to apply the transit-time theorem to the equivalent thin lens of such fields. In doing so, information is readily obtained about magnification and optical constants of such systems, without knowledge of the trajectory. The accuracy of the results depends upon the number of terms considered in a series expansion. Only first-order approximations for electric and magnetic lenses and for plane accelerating fields are presented in this paper. Higher-order approximations may be obtained by the same method.

The purpose of this paper is not so much to supplement prior information as to present a method that may prove useful in the design and understanding of electron-optical instruments.

II. The Thin Lens

Let us consider the arrangement shown in Fig. 1, in

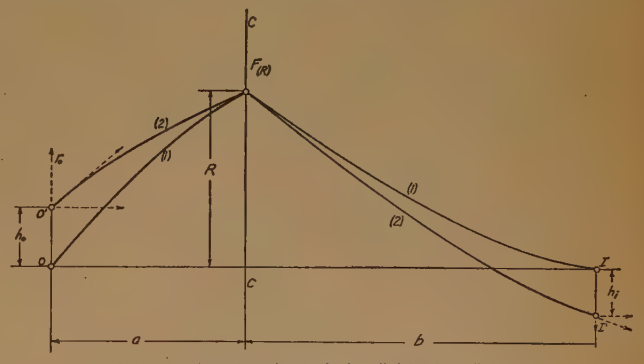


Fig. 1—Derivation of the "thin-lens" law.

which the object OO' of height h_0 is a source of mass points of mass μ which are emitted at various angles with radial velocities $\dot{r}_0$ and which are accelerated along the axis. We suppose that no radial forces act upon the particles until they reach the plane C which they strike at a distance R off the axis. In the plane C, we assume that a finite impulse $I(R) = F \cdot \tau$, directed toward the axis, acts upon the particle. τ is the transit time in the "lens" C. This impulse is a function of the radial distance alone, and is independent of the angle at which the particle arrives at C.

We attempt to establish the conditions under which all particles originating from any point in the object plane converge to a single point on the image plane II'. Since there is no radial force in the space either in front

of or behind the "lens" C, the radial component of motion may readily be written in the following form:

$$h_0 + \dot{r}_0 T_0 = R \tag{1}$$

$$h_i = R + \dot{r}_i T_i. \tag{2}$$

In these equations, T_0 and T_i represent the transit times required for the mass points to traverse their paths in the object and image spaces respectively. The radial velocities $\dot{r}_0$ and $\dot{r}_i$ remain unaltered during the path outside the lens. The radial impulse acting on a particle in the plane C causes a change in the momentum of the particle given by

$$\mu(\dot{r}_i - \dot{r}_0) = -I. \tag{3}$$

(The impulse I is taken as positive, if directed towards the axis.) By adding (1) and (2) and combining the sum with (3), we obtain

$$R[1/T_0 + 1/T_i] - [h_i/T_i + h_0/T_0] = I/\mu. \tag{4}$$

We now define the image distance b as the distance between the lens C and the point I, where a particle emitted from O on the axis, having passed through the object distance a and the lens C, reaches the axis again. For any such ray (1 in the figure), the conditions are by definition

$$h_0 = 0 \quad \text{and} \quad h_i = 0. \tag{5}$$

Substituting (5) into (4), we find that particles originating from and meeting on the axis obey the following form of (4):

$$R[1/T_0 + 1/T_i] = I/\mu. \tag{4a}$$

In order for the point I to be the image of O, it is necessary that all particles emerging from O, regardless of their different values of radial velocity, must meet again at I. Since we have assumed the object and image spaces to be free of radial forces, various initial velocities $\dot{r}_0$ must mean different values of R. Hence the condition for the particles to meet again at I is that the transit time T_i must be independent of R. This is satisfied only if the impulse I (R) is a linear function of R. We introduce, accordingly, a time constant T_f such that

$$I/\mu = R/T_f. \tag{6}$$

It is seen immediately that (4a) becomes

$$\boxed{1/T_0 + 1/T_i = 1/T_f}. \tag{7}$$

This is the fundamental equation for thin-lens action upon corpuscular rays. T_f turns out to be the focal transit time, and its value on either side of the lens is equal to that on the opposite side. It has been shown that the "thin-lens" formula for the mechanical model is a relation between transit times rather than between distances as encountered in elementary optics.

It remains to be shown that mass points which start from an object point O' not on the axis meet in a corresponding image point I'. Let us consider that ray 2 in Fig. 1 is such a path starting from a point O', h_0 off the axis, and reaching the lens C at a distance R off the axis. To find the impulse I which acts upon that particle within the lens we must remember that the impulse does not depend upon the angle of incidence, but depends solely upon the distance R. Accordingly, we may find the impulse for ray 2 readily from (4a) which holds for ray 1. If we still keep in mind that the two transit times T_0 and T_i are identical for both rays 1 and 2 in the spaces outside the lens, then it follows that (4a) must hold in combination with (4), and that they have the group of values R, T_0, T_i, and I in common. We find, therefore, by subtraction of (4a) from (4)

$$h_i/T_i + h_0/T_0 = 0. \tag{4b}$$

Since this equation no longer contains R, it follows that all rays emerging from O' recombine at I'. Equation (4b) gives immediately an expression for the ratio of the distances $|h_i|$ and $|h_0|$, which is the magnification of the system

$$\boxed{m = T_i/T_0.} \tag{8}[11]$$

This is the transit-time theorem of electron-optical magnification, a formula which proves to be of great practical value, as will be discussed later.

For corresponding points on the axis, having radial velocities $\dot{r}_{00}$ and $\dot{r}_{i0}$ respectively, we obtain $\dot{r}_{00}T_0 = R = -\dot{r}_{i0}T_i$. In this case the transit-time theorem yields another definition of magnification:

$$m = -\dot{r}_{00}/\dot{r}_{i0}. \tag{8a}$$

This is a form to which we shall refer later in the theory of thick lenses.

By combining the lens formula (7) and the magnification formula (8) one finds that

$$T/T_f = (m+1)^2/m \tag{9}$$

where $T = T_0 + T_i$ represents the total transit time from object to image. Since the expression on the right has a minimum value for $m=1$, it is apparent at once that unit magnification results in the "shortest possible tube," that is, a system for which the total transit time is equal to four times the focal transit time. It will be shown that this is also true in the general case of extended fields.

III. Application of the Transit-Time Theorem

The transit-time theorem has been derived from a purely mechanical standpoint. Although most electron-optical instruments operate with distributed electric or magnetic fields, extended too much along the axis to be considered as thin lenses, it is nevertheless quite useful to apply the transit-time theorem to such instruments in order to obtain an approximation of the magnification to be expected. Such application is possible as soon as the position of an equivalent thin lens is defined reasonably well. To find the transit times for a given potential distribution, the following reasoning is used.

If $V(O,z)$ is the potential distribution along the axis, the transit time for an electron accelerated by V to traverse the distance $z - z_0$ is

$$T = \frac{1}{\beta}\int_{z_0}^{z} \frac{dz}{\sqrt{V_{(0,z)}}} \tag{10}$$

where $\beta = \sqrt{2\epsilon/\mu} = 5{,}95 \cdot 10^7$ centimeter per second volt$^{1/2}$.

[11] In $m = -h_i/h_0$, the negative sign indicates inversion of the image.

In the special case of a linear potential distribution, this integral reduces to

$$T = 2(z - z_0)/\sqrt{V_{(z)}} + \sqrt{V_{(z_0)}} \cdot 1/\beta. \qquad (10a)$$

It is a general property of circular symmetry that the potential $V(r, z)$ at any point in the field may be expressed in terms of the potential along the axis $V(O, z)$ and its even derivatives:[12]

$$V_{(r,z)} = V_{(0,z)} - (r^2/2^2) \cdot V_{(0,z)}'' + r^4/2^2 \cdot 4^2 \cdot V_{(0,z)}^{(4)} - \cdots . \qquad (11)$$

Consequently, the radial component of force acting upon a particle of charge ϵ becomes

$$F_r = \epsilon \cdot \partial V/\partial r = -\epsilon r[2 \cdot V_{(0,z)}''/2^2 - r^2 \cdot 4 V_{(0,z)}^{(4)}/2^2 \cdot 4^2 + \cdots + (-1)^n \cdot r^{2n-2} 2n \cdot V^{(2n)}/2^2 \cdot 4^2 \cdots (2n)^2] \qquad (12)$$

where $V_{0z}^{(2n)}$ stands for the $2n$th derivative of the potential V_{0z} along the axis. From (12) we deduce that the object and image spaces may be considered to be free from radial impulses provided that their potential distribution along the axis is a linear function[13] of z. In practice, such systems, known as electron accelerators, are frequently encountered. They consist either of a series of metallic rings along which a constant-voltage gradient is maintained by bleeders, or of a graphite coating in the form of spirals through which an adequate current flows, producing a linear voltage drop. It follows from (12) that such linear accelerators do not exhibit lens action. They do, however, influence the electron-optical magnification noticeably, as will be observed in the following.

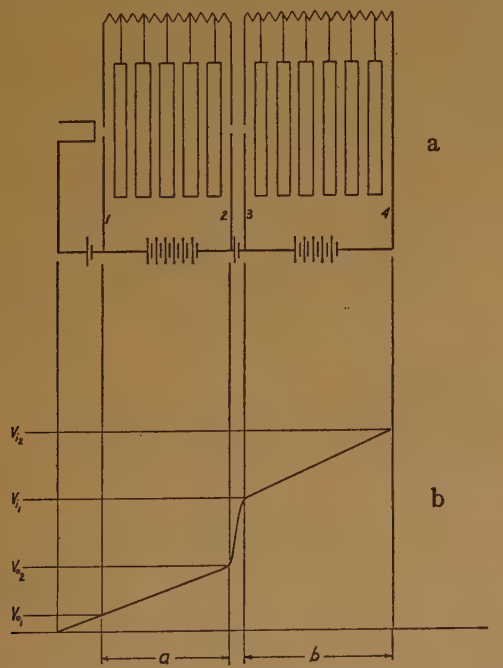


Fig. 2—Cathode-ray tube with accelerators and electrostatic lens.

Let us now consider Fig. 2a, which shows an electrostatic system that is representative of several practical devices. This system consists of a lens region, confined between the adjacent diaphragms 2 and 3, and two electron accelerators, which form the object and image spaces confined between the diaphragms 1 and 2, and 3 and 4. The four diaphragms are at the potentials V_{01},

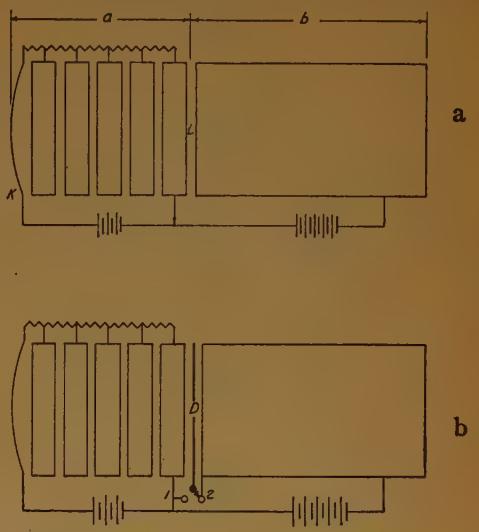


Fig. 3—Image tube with a, fixed magnification and b, variable.

V_{02}, V_{i1}, and V_{i2} as noted in Fig. 2b. The potential gradient in the two spaces is made constant over most of the electron path. Under the assumption that the nonlinearity of the potential is more or less confined to the region 2–3, the transit-time theorem (8) may be applied directly by considering the object and image transit times to be given according to (10). Therefore, it is found that

$$m = b/a\sqrt{V_{01}} + \sqrt{V_{02}}/\sqrt{V_{i1}} + \sqrt{V_{i2}}. \qquad (13)$$

This yields the electron-optical magnification of any device of the type of Fig. 2a with an equivalent thin lens acting between four electrodes at predetermined potentials.

One such device is Zworykin's "image tube."[14] This instrument is shown schematically in Figs. 3a with fixed magnification and in Fig. 3b with variable magnification. In both forms a ring accelerator extends over the distance a from the photocathode K to the lens L, while the image space consists of a uniformly coated wall and may be considered field-free. The lens action in Fig. 3a is brought about by a potential difference maintained between the boundary diaphragms of the object and image spaces, and is assumed to be ideal for paraxial rays (6). In the tube in Fig. 3b, a special diaphragm D is arranged between the object and image spaces so that it can be connected to either one of them. Applying (13) to these systems, we must set $V_{01} = 0$ and $V_{i1} = V_{i2} = V_i$, the latter being true because of the uniform coating along the wall of the image space. Hence it follows that

$$m = b/2a\sqrt{V_{02}/V_i}. \qquad (13a)$$

In the system in Fig. 3a, it is found that the potential

[12] L. Marton and R. G. E. Hutter, "The transmission type of electron microscope and its optics," Proc. I.R.E., vol. 32, pp. 3–12; January, 1944.

[13] This accounts for parabolic trajectories in the arrangement of Fig. 1.

[14] V. K. Zworykin and G. A. Morton, "Applied electron optics," *Jour. Opt. Soc. Amer.*, vol. 26, pp. 181–189; 1936.

difference between the two spaces is very small compared with V_i, and hence the magnification is very nearly $b/2a$. It is apparent that the magnification of the system in Fig. 3b, however, is greater than $b/2a$ when the switch is in position 2, (that is, when the diaphragm D is connected to the image space), since the transit time in the object space is smaller, and is less than

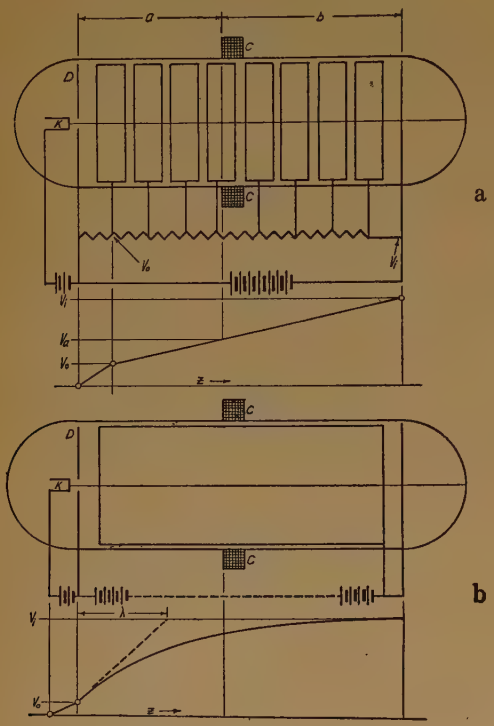


Fig. 4—Cathode-ray tubes with combined electric and magnetic fields.

$b/2a$ when the switch is in position 1. Intermediate values are obtained when the diaphragm is connected to a potentiometer between 1 and 2.

To give yet another illustration of the usefulness of transit-time theorem, we consider the two tubes indicated in Figs. 4a and 4b. In both cases the lens action is brought about by a short or shielded magnetic coil C, which is to focus the image of the first diaphragm D on the screen S. Both tubes are of the same length, but the tube shown in Fig. 4a acts with constant acceleration over its entire length, whereas the one shown in Fig. 4b has a uniform coating so that the electrons traverse most of their path at a constant maximum speed. As will be shown later, the two principal planes of short coils coincide with the plane of symmetry of the windings, or the gap in the magnetic shield, provided that the speed of the electron is not subject to too great an alteration in the lens. The magnification may be estimated, therefore, not only in the case of constant field outside the lens, but even in cases of nonlinear potential distribution (as in Fig. 4b), so long as the electrostatic-lens action introduced by such nonlinear fields remains negligible compared with that of the magnetic lens.

In Fig. 4a the electrons enter at the diaphragm D with a speed equivalent to V_0 volts and because of a homogeneous coating on the tube wall move with constant acceleration to the screen, which they reach with a speed equivalent to V_i volts. In this case, the only radial impulses are produced by the coil C. The potential V_a at the lens at $z=a$ may be written

$$V_a = V_0(b/a + b) + V_i(a/a + b) \tag{14}$$

and, by (13), the magnification is

$$m = m_0 \frac{1 + \sqrt{V_0/V_i \cdot (m_0 + 1)/1 + m_0(V_0/V_i)}}{1 + \sqrt{(m_0 + 1)/1 + m_0\,(V_0/V_i)}} \tag{15}$$

where $m_0 = b/a$ is the "optical" magnification of the system.

In Fig. 5a, m is plotted against m_0 for different values of $V_0 : V_i$. It is seen that a considerable reduction of the spot on the screen may be obtained by decreasing V_0. With a total tube length of 36 centimeters, for instance, of which only 4 centimeters is given to the object space, the "optical" magnification would be $m_0 = 8$, whereas the electronic magnification turns out to be one fourth of that value by reducing the potential of the diaphragm to 1 per cent of that of the screen. With a screen potential of 5000 volts and a diaphragm bias of 50 volts, for instance, the resolving power of the system in Fig. 4a can be increased four times over that of a tube with constant speed.

In the system shown in Fig. 4b, the electrons reach their maximum speed after traversing the short distance λ along the axis. The potential distribution along the axis is very nearly exponential and may be expressed as

$$V_{(z)} = V_i(1 - (V_i - V_0/V_i)e^{-z/\lambda}). \tag{16}$$

The slope constant λ may be determined experimentally to be of the order of magnitude of one fourth of the tube

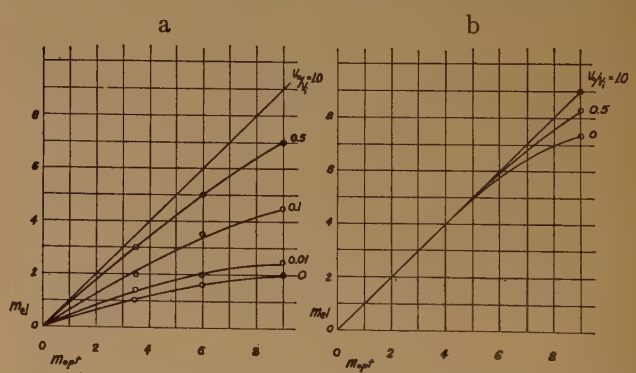


Fig. 5—Electronic magnification of the tubes shown in Fig. 4.

diameter. The transit time for an electron to traverse l centimeters in the field defined by (16) follows from (10):

$$T_l = T_{\min}\,[1 + (\lambda/l)\ln(2/1 + V_0/V_i)]. \tag{17}$$

Apparently, this transit time is not much longer than it would be for electrons traveling over the whole tube length with maximum velocity. This would require a transit time of

$$T_{\min} = 1/\beta(l/\sqrt{V_i}). \tag{18}$$

As a matter of fact, the actual transit time may be

written in terms of T_{min} as follows:

$$T = T_{min}(1 + 0.7\lambda/l) \quad (17a)$$

or

$$\Delta T/T = 0.7\lambda/l. \quad (17b)$$

This means that the actual transit time, taking into account the small initial velocity, is only a few per cent larger than it would be in the absence of the low-voltage region near the diaphragm D. Consequently, the "electronic" magnification cannot be expected to differ greatly from the "optical" magnification b/a. By application of the transit-time theorem,

$$m = \frac{1 + \lambda/b \ln (2/1 + V(z)/V_i)}{1 + \lambda/a \ln (1 + V(z)/V_i/1 + V_0/V_i)}. \quad (19)$$

This has been plotted in Fig. 5b for the same tube length and lens position as in Fig. 5a, that is, with the dimensions $l=36$; $\lambda=1.2$ centimeters. It is noted that the influence of the primary voltage is much smaller than in the previous case of constant acceleration. The reduction in spot size is always less than 20 per cent, even with negligible speed in the object space. Hence, it is seen that reduction of the voltage in the object plane does not result in an efficient reduction of spot size on the screen for constant-speed systems, but only for tubes with long electron accelerators. Whether the use of the latter is justified is a matter of production costs.

IV. Mathematical Theory of the "Thick" Lens

A study of the motion of electrons in extended axially symmetric fields is to be undertaken next. Let us consider the path 1 in Fig. 6 in which an electron starts at the point O on the axis with a radial velocity $\dot{r}_0$ and finally reaches the point I on the axis. The electron is accelerated along the axis z by an electric field with the potential $V(z)$ and is subjected to the radial forces $F(r)$ throughout its entire path.

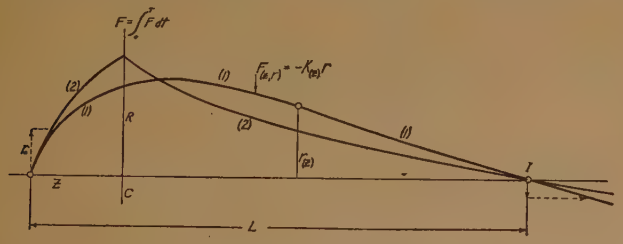


Fig. 6—Derivation of "thick-lens" law.

The only assumption necessary regarding $F(r)$ is its proportionality to the distance r off the axis at all points along the path. This restricts us, practically, to paraxial rays. Accordingly, the radial component of force is expressed as

$$F(r) = -K(z)\cdot r \quad (20)$$

in which $K(z)$ is the force exerted on the particle at unit distance off the axis, and may be called the "deflection constant." K depends explicitly upon z and is positive so long as the force F is directed toward the axis. The equations of motion then become

$$\mu\ddot{r} = -K_{(z)}\cdot r \quad (21a)$$

and

$$\mu\ddot{z} = \epsilon\cdot dV_{(0,z)}/dz. \quad (21b)$$

The latter equation may be integrated to give

$$dt = dz/\sqrt{2\epsilon/\mu}\cdot\sqrt{V_{(0,z)}} = dz/\beta\sqrt{V_{(0,z)}}. \quad (22)$$

This will enable us to transform time integrals into path integrals.

The usual method of procedure is to eliminate t from (21a) and (21b), in order to establish a differential equation for the trajectories. We choose a different way of computation, however, which will give us the desired information about the rays in the vicinity of the object and image directly, without dealing with the trajectory problem at all. To this end, let us write the integral of (21a) in the following form:

$$\dot{r}(t) = -\frac{1}{\mu}\int_0^t K(z)\cdot r dt + \dot{r}_0. \quad (23)$$

We separate K, which depends on z alone, from the trajectory $r(t)$, by successive partial integration:

$$\int_0^t Krdt = r\int_0^t Kdt - \dot{r}\int_0^t dt\int_0^t Kdt + \ddot{r}\int_0^t dt\int_0^t dt\int_0^t Kdt - \cdots. \quad (24)$$

The result is the following series development in terms of successive derivatives of the trajectory with respect to time:

$$\dot{r}(t) - \dot{r}_0 = -rQ_1(t) + \dot{r}Q_2(t) - \ddot{r}Q_3(t) + \cdots = \sum_{n=1}^{n}(-1)^n\frac{d^{n-1}r}{dt^{n-1}}Q_n. \quad (23a)$$

The coefficients Q_n are defined as follows:

$$Q_1 = \frac{1}{\mu}\int_0^t Kdt$$
$$Q_2 = \frac{1}{\mu}\int_0^t dt\int_0^t Kdt = \int_0^t Q_1 dt \quad (25)$$
$$Q_n = \int_0^t Q_{n-1}dt$$
$$Q_m(t=0) = 0 \qquad (m = 1, 2, \cdots, n).$$

These are, obviously, the integrals of deflection with respect to time. By means of (22), they may be transformed into the following integrals with respect to z:

$$Q_1(z) = -\frac{1}{\beta}\cdot\int_0^z \frac{K/\mu\cdot dz}{\sqrt{V(z)}}$$
$$Q_n(z) = -\frac{1}{\beta^n}\int_0^z\frac{dz}{\sqrt{V}}\cdot\int_0^z\frac{dz}{\sqrt{V}}\cdots\int_0^z\frac{K/\mu}{\sqrt{V}}dz. \quad (25a)$$

It is to be noted that these integrals have the dimensions

$$[Q_n(z)] = \sec^{n-2}. \quad (26)$$

One observes, especially, that Q_2 is a pure number. It is evident from (25a) that if $K(z)$ is known, all these quadratures may be evaluated without difficulty in any practical case, at least by graphical methods.

The expression (23a) for the radial velocities is in the form of an infinite series of functions of t, in which each term is a product of the $(n-1)$st derivative of r, taken at t, and the definite integral Q_n, taken between zero and t. It is shown in the Appendix, that this series is convergent, provided that $r_{(t)}$ is analytic at least over the range $-\epsilon<t<2t+\epsilon$; $\epsilon>0$ where T is the total transit time between object and image. Furthermore, the maximum absolute value of the terms is shown to decrease at least as $1/n$ or faster than $1/n$. Under these conditions, first-order approximations may be obtained, within the range in question, by breaking the expansion (23a) off after the second term. Better approximations may be obtained, by considering subsequently further terms. However, the first-order approximation is already in good agreement with experimental results in cases of sufficiently strong fields. By breaking (23a) off at $n=2$, we obtain

$$\dot{r}(1) = \frac{\dot{r}_0 - r(l)Q_1(l)}{1 - Q_2(l)}. \tag{28}$$

If we consider an electron reaching the axis at $z=l$, $r=0$, after a total transit time $t=T$, we obtain

$$\dot{r}_0/\dot{r}(l) = 1 - Q_2[z = l]. \tag{29}$$

It is seen, therefore, that the ratio of the radial velocities at the two intersections with the axis is the same for all rays, since Q_2 is taken along the axis rather than along any individual path. If the given distributed field were replaced by an equivalent "thin lens," located at C in Fig. 6 the same ratio would result for the hypothetical path 2 as for the actual path 1 and it would be equal to the ratio of transit times in front of and behind the lens, and hence to the magnification. Hence, for a thick electron lens, the transit-time theorem (8a) yields

$$\boxed{m = Q_2(l) - 1} \tag{30}$$

that is, the electronic magnification of a thick lens is equal to the second-order integral of the deflection with respect to the total transit time, minus 1. It will be shown later that this theorem makes it possible to find the picture size for any given image position without even knowing the object position. To find the latter, a second integration of (23a) is necessary:

$$r = \dot{r}_0 t + \sum_{n=1}^{\infty} (-1)^n \cdot n \cdot r^{(n-1)} Q_{n+1}. \tag{31}$$

This series is convergent because of the convergence of (23a). By breaking the expansion off after $n=2$, we obtain for $r_{(t)}$

$$r_{(t)} = \frac{\dot{r}_0 t + 2\dot{r}Q_3(t)}{1 + Q_2(t)} \tag{32}$$

and, if the ray is to intersect the axis, $(r(T)=0)$ after the total transit time $t=T$, this reduces to

$$-(\dot{r}_0/\dot{r}_i) = 2Q_3(T)/T. \tag{33}$$

By combining this with (30), there results

$$T = 2Q_3(l)/Q_2(l) - 1. \tag{34}$$

It will be shown in the following section that (30) and (34) are necessary and sufficient to find both the magnification and positions of the corresponding object and image planes of the system. The position of the equivalent thin lens may also be found on the transit-time curve by making use of the following relation, which results from (30) and (8):

$$T_0 = T/Q_2(l). \tag{35}$$

V. The Electrostatic Lens. The Hyperbolic Diagram

Let us consider an electrostatic field, described by a potential plot $V(r, z)$ as obtained with the aid of an electrolytic trough. This field may be expressed in terms of its potential distribution along the axis $V(0, z)$, as shown in (11). Then, from (12), one finds the radial component of force for paraxial electrons

$$F_r = -(\epsilon/2)V_0'' \cdot r \tag{36}$$

and also the "deflection constant" K as defined in (20)

$$K_{(z)} = (\epsilon/2)(d^2V/dz^2). \tag{20a}$$

Inserting this into (25), we find the integral coefficients

$$Q_n = \frac{1}{4}\beta^{2-n} \int_0^z \frac{dz}{\sqrt{V}} \int_0^z \frac{dz}{\sqrt{V}} \cdots \int_0^z V'' \frac{dz}{\sqrt{V}}. \tag{25}$$

Q_2 turns out to be independent of β. The magnification (30) is therefore also independent of β. In the expressions for the transit time, (34) and (10), both sides are found to have the factor $1/\beta$ in common. This indicates that the optical constants are independent of the nature of the charged particles. They may be found, therefore, by using these four integral coefficients:

$$Q_0 = \int_0^z \frac{dz}{\sqrt{V}} \quad \frac{\text{cm}}{\text{volt}^{1/2}} \tag{37a}$$

$$Q_1 = \int_0^z \frac{\frac{1}{4}V''}{\sqrt{V}}\, dz \quad \frac{\text{volt}^{1/2}}{\text{cm}} \tag{37b}$$

$$Q_2 = \int_0^z Q_1 \frac{dz}{\sqrt{V}} \tag{37c}$$

$$Q_3 = \int_0^z Q_2 \frac{dz}{\sqrt{V}} \quad \frac{\text{cm}}{\text{volt}^{1/2}} \tag{37d}$$

To illustrate the use of the new method of calculation we treat the problem of the "electron gun." The entire procedure is indicated in Fig. 7 in successive steps. Fig. 7a shows the constructional details of the system, and Fig. 7b indicates the axial potential distribution, obtained with the aid of an electrolytic trough. The plot was taken for $V_0=1000$ volts and $V_i=4600$ volts, but the behavior of the system for any arbitrary gun voltage V_g may be found by adding the following voltage v to the original values of the plot

$$v = V_i(V_g - V_0)/(V_i - V_g). \tag{38}$$

This has been done in the diagram for three values of gun voltages. ($V_g=1000$, 750,500 volts; $v=0$, -300 and -550 volts, respectively.)

The first integral Q_1, obtained from this potential distribution, either by graphical integration or by an

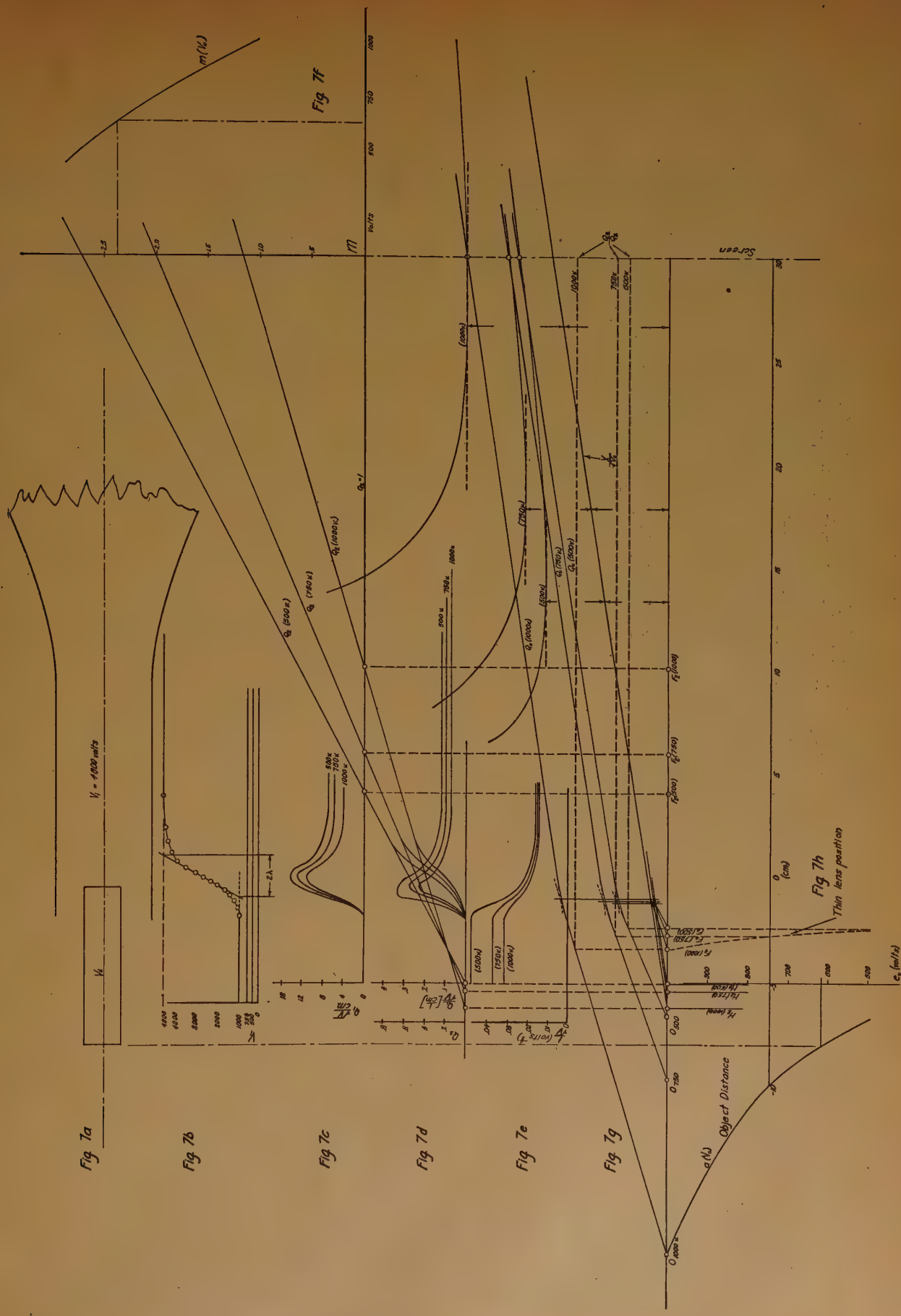


Fig. 7—The hyperbolic diagram for the determination of optical constants of a thick lens.

analytical method, (Appendix B) has been plotted for various voltages in Fig. 7c. Q_1, which is recognized as the radial impulse, has the value zero in the object plane, rises to a maximum at the exit of the gun, and then falls to a constant value after leaving the lens. This reflects the presence of both positive and negative lens action in electron guns.

The second integral of deflection Q_2 is readily obtained from Q_1 by graphical integration according to (37c), and is shown in Fig. 7d. Q_2 gives us immediate information about four optical constants of the system: (1) the second focal point F_2, (2) the second principal plane H_2, (3) the second focal length f_2, and (4) the magnification m. This is so because of the following reasoning.

Due to the fact that Q_1 is a constant in the image space, the graphical representation of Q_2 versus distance eventually becomes a straight line with the slope $Q_{1(I)}/\sqrt{V_i}$, as seen in the graph Fig. 7d. We prolong this line backward until it intersects the axis at a point H_2 ($z=z_h$), and the line at unit distance parallel to the axis at a point above F_2 ($z=z_f$). It is easy to see that the latter intersection yields the second focus of the system, since the magnification there becomes zero (equation (30)) and the total transit time becomes infinite (equation (34)). The fact that the intersection of Q_2 with the axis gives the position H_2 of the second principal plane may likewise be proved in an elementary manner.

Hence, the second focal length is

$$f_2 = \frac{\sqrt{V_i}}{Q_{1(I)}} = \frac{4\sqrt{V_i}}{\int_0^l V''dz/\sqrt{V}} \tag{39}$$[15]

and $$Q_{2i} = Q_{1i}/\sqrt{V_i}\cdot(z - z_H) = (z - z_H)/f_2. \tag{40}$$

Fig. 7f shows how the magnification is obtained from the graph for any arbitrary image plane and voltage ratio. According to (30), the magnification m can be read numerically from the intersection of Q_2 with the screen plane. Equation (30) furnishes the following expression for the magnification of a thick electrostatic lens:

$$m = \frac{1}{4}\int_0^l \frac{dz}{\sqrt{V}}\cdot\int_0^l \frac{V''dz}{\sqrt{V}} - 1 \tag{41}$$

an expression which is invariable with respect to the multiplication of all potentials by the same constant factor. Thus, the magnification remains unaltered so long as the ratio V_i/V_0 is constant. Furthermore, it does not depend on the e/m ratio: the lens acts for ions as well as for electrons. Finally, the magnification is found directly from the image distance, even before the corresponding object distance is known.

[15] In general, (39) is not the equivalent of the graphical construction as outlined in Fig. 7d. It anticipates the result correctly only when $Q_2(z)$ is already a straight line, before intersecting the unit horizontal. This holds for strong fields and relatively short lenses. For weak fields however, (39) cannot replace the graphical construction, which is usually more accurate than the formula. The latter may be inaccurate to about 50 per cent. (See G. N. Plass, "Electrostatic electron lenses with a minimum of spherical aberration," *Jour. Appl. Phys.* vol. 13, no. 1, pp. 50–51; January, 1942.)

To find the object distance, it is necessary to use (34) for the total transit time. This equation may be interpreted geometrically as an intersection between the following two curves:

$$y = 2Q_3/(Q_2 - 1) \tag{42a}$$

$$y = Q_0 + \text{constant}. \tag{42b}$$

If we introduce a new reference system in which all distances x are measured from the second principal plane ($x=z-z_h$), we may write Q_2 from (40),

$$Q_2 = x/f_2. \tag{40a}$$

The integration $Q_3=\int Q_2(dz/\sqrt{V})$ may now be carried out formally, and one obtains the form for the curve y from (42a)

$$y = (1/\sqrt{V_i})[x(x - \xi)/(x - f_2)] \tag{43}$$

in which the small length ξ has the value

$$\xi = \frac{1}{2}f_2 - \int_0^z \sqrt{\frac{V_i}{V_z}}Q_2dz. \tag{44}$$

This is a small correction factor, which is negligible in many special cases.

Equation (43) is for a hyperbola which has the asymptotes

$$y = f_2 \tag{45a}$$

$$y = x/\sqrt{V_i}. \tag{45b}$$

In Fig. 7g, three of these hyperbolas for different gun voltages have been drawn. These hyperbolas all have the asymptote (456) in common, as shown.

Hence, in order to obtain any pair of corresponding image and object planes, it is necessary to proceed as follows: Find the intersection of the hyperbola (43) with the screen plane $z=z_i$, and draw the transit-time curve

$$y = Q_0 = \int_0^z \frac{dz}{\sqrt{V(z)}} \tag{37a}$$

through this point. This, in turn, intersects the axis at the desired object abscissa, and vice versa. Any pair of corresponding planes may be determined by shifting the Q_0 curve vertically and finding its intersections with the axis and the hyperbolas respectively. By doing this for several gun voltages, the relation between object distance and voltage is readily found, as shown in Fig. 7g. The results are in good agreement with the experimental data of the literature.[16]

There are four special cases to be emphasized:

(1) Screen at F_2. $z_i=z_f$. The geometrical construction leads to the fact that the object is located at infinity, that is, that F_2 is indeed the second focus of the system.

(2) Screen at infinity. $z_i=\infty$. The asymptote (456) has the same slope as the branch of the transit-time curve (37a) in the image space. By bringing the latter into coincidence with the asymptote, the first focal point F_1 is obtained by the intersection with the axis. This is shown in Fig. 7 for several gun voltages.

(3) The shortest possible tube. If the screen is at twice the focal distance from the second principal plane H_2,

[16] See Section 6.6 of footnote reference 2.

$x = 2f_2$, then by (43) the hyperbolas, as well as the total transit time, are at a minimum. The magnification is unity. Hence, for any magnification different from unity the tube is longer than the shortest possible tube, the length of which still depends upon the voltage ratio.

(4) The equivalent thin-lens position is readily found from (35) and has been constructed in Fig. 7h. It is found to fall short of the first focal point F_1, which it approaches more and more as the screen is further removed. Its position is nearly constant with respect to the gun voltage, however, and is found to be fixed at about 1.5 gun diameters inside the gun, in excellent agreement with measurements.[17]

Thus, to summarize briefly, all optical qualities of a given axially symmetric field may be determined analytically, simply by evaluation of the first three integrals of deflection with respect to time. Of these, Q_1 gives the focal length, Q_2 the magnification, and Q_3 the positions of image and object in the form of an hyperbolic diagram, which may easily be drawn for all voltage ratios.

VI. The Magnetic Lens

With the same method outlined in the preceding section, the optical constants of extended magnetic fields may be determined by simple graphical integration, as soon as the magnetic-field strength H_0 along the axis is known. This may be determined readily with the search-coil method. In some of the simpler cases, H_0 may also be computed theoretically.

As is well known, the transverse component of a combined electrostatic and magnetic field exerted on paraxial electrons is given by

$$F_r = -(\epsilon r/2)[V'' + (1/2)(\epsilon/\mu)H^2] \qquad (46)$$

and is thus the sum of an electrostatic action, due to the curvature of the equipotential surfaces (treated in the preceding section), and of a magnetic "lens" action. The "deflection constant" K of the latter alone follows from the definition (20):

$$K_{(z)} = -(\epsilon^2/4\mu)\cdot H^2 \qquad (20b)$$

and the integral coefficients become

$$Q_1 = \frac{1}{16}\beta^3 \int_0^z H^2 \frac{dz}{\sqrt{V}};$$

$$Q_2 = \frac{1}{16}\beta^2 \cdot \int_0^z \frac{dz}{\sqrt{V}} \cdot \int_0^z H^2 \frac{dz}{\sqrt{V}}; \quad \left[\beta = \sqrt{\frac{2\epsilon}{\mu}}\right];$$

$$Q_3 = \frac{1}{16}\beta \cdot \int_0^z \frac{dz}{\sqrt{V}} \cdot \int_0^z \frac{dz}{\sqrt{V}} \cdot \int_0^z H^2 \frac{dz}{\sqrt{V}};$$

$$Q_n = \frac{1}{16}\beta^{4-n} \int_0^z \frac{dz}{\sqrt{V}} \cdots \int_0^z \frac{dz}{\sqrt{V}} \cdot \int_0^z H^2 \frac{dz}{\sqrt{V}}.$$

The expression for Q_2, which is proportional to ϵ/μ informs us that any increase in potential must be compensated for by a reinforcement of the magnetic field of the magnitude of the square root of the increase, if the magnification is to be kept constant and the image kept in focus. The remainder of the analytical procedure is the same as in the electrostatic case, as is also the hyperbolic diagram and its use for the determination of object and image planes and magnification.

For example, the field distribution $H^2(z)$ and the first two integrals for a short coil have been drawn in Fig. 8.

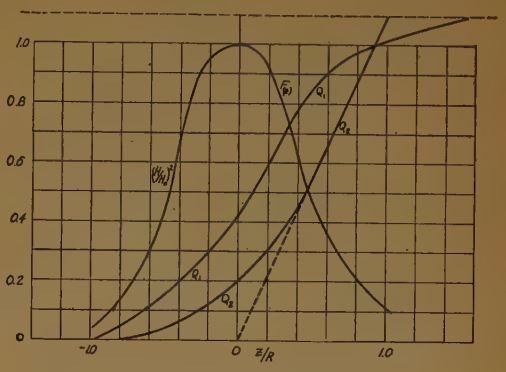


Fig. 8—Proof that the circular magnetic ring acts as a "thin lens."

It has been assumed that the field distribution is identical with that produced by a single circular loop of radius ρ. Thus

$$(H/H_0)^2 = 1/[1 + (z/\rho)^2]^3. \qquad (47)$$

By carrying out the first two integrations graphically, under the assumption of constant electron speed under the lens, it is found that the straight-line portion of Q_2 intersects the axis in the plane of symmetry of the coil. This proves, therefore, that for such a short coil, the two principal planes coincide with the plane of symmetry and with each other. This justifies the treatment of such coils as "thin lenses," as has been done in Part I of this paper.

VII. The Plane Accelerating Field

We wish to demonstrate how the method of the integrals of deflection may be applied to a case in which no real image, but only a virtual one, is formed by an axially symmetric field. We consider the motion of electrons emanating from the cathode of a cathode-ray tube under the influence of a field described by

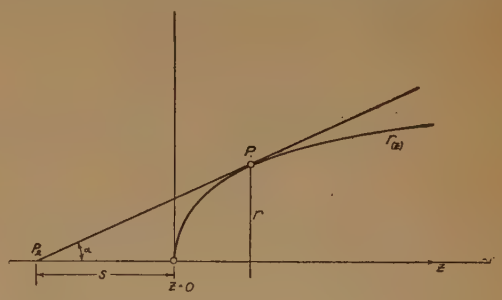


Fig. 9—Electron trajectory in plane accelerating field.

$$V_{(z)} = V_0 \cdot (z/z_0)^n \qquad n < 2. \qquad (48)$$

Langmuir's law is an example:

$$V^{3/2} = z^2 I/C; \qquad C = 2.33 \times 10^{-6}$$
$$V_{(z)} = V_0 \cdot (z/z_0)^{4/3}. \qquad (48a)$$

In Fig. 9, let $z = 0$ be the cathode surface, and $r(z)$ the electron trajectory. The tangent to the latter at point P

[17] See pp. 110–117 of footnote reference 2.

may intersect the axis at a point P_2 at a distance s from the cathode. Choosing s as negative, if P_2 is behind the cathode surface, we find from Fig. 8 that

$$s = z - r/\tan\alpha = z - (r/\dot{r})\beta\sqrt{V_{(z)}}. \quad (49)$$

For $r(z)$ and $\dot{r}(z)$ we may use (28) and (32), each of which contains the initial radial speed $\dot{r}_0$. Combining these with (49), we obtain

$$s = z - \sqrt{V_{(z)}}\,\frac{(1-Q_2)Q_0 + 2Q_3}{1 + Q_2 - Q_0Q_1}. \quad (50)$$

We thus find that $\dot{r}_0$ and β have canceled out. All rays, therefore, seem to originate from one and the same spot P_2, which is the "virtual image" of the emitting center at the cathode surface.[18]

By carrying out the integrations, it is found that, although Q_0 remains finite, Q_1, Q_2, and Q_3 become infinite for $z=0$, so long as $n<2$. The ratios $Q_2:Q_1$ and $Q_3:Q_1$ remain finite, however, and we evaluate them by extending the integration up to a boundary δ near the cathode and then determining $\lim_{\delta\to 0} Q_n/Q_1$. Thus

$$\lim_{\delta\to 0} Q_2/Q_1 = 2/(2-n)\cdot z/\sqrt{V_{(z)}};$$
$$\lim_{\delta\to 0} Q_3/Q_1 = 2/(2-n)^2\cdot z^2/V_{(z)}. \quad (51)$$

It is to be noted that

$$\lim Q_3/Q_1 = 1/2[\lim Q_2/Q_1]^2.$$

Combining these relations with (49), one finds

$$s = z - \sqrt{V_{(z)}}\lim Q_2/Q_1 = -n/(2-n)\cdot z. \quad (52)$$

That is, the virtual image is located behind the cathode for all potential distributions which increase with less than the second power of z.

To find the actual trajectories in such fields all that is necessary is to write down the geometrical expression of the fact that all rays seem to emerge from the same point P_2, as defined by (52):

$$-s + z = [r/(dr/dz)]; \quad dr/dz = (2-n)/2\cdot r/z. \quad (53)$$

Hence, the form of the trajectories follows immediately:

$$r = \text{constant } z^{1-n/2}. \quad (54)$$

In the case of Langmuir's space-charge law, for example, with $n=4/3$ the electrons describe cubic parabolas:

$$r = \text{constant } \sqrt[3]{z} \quad (55)$$

and the virtual image of the center of emission is always twice the distance behind the cathode, that the plane of the observer is in front of it.

In the case under consideration, the electrons, which arrive in the plane $z=z_0$, appear to have emerged from a plane at $z=-2z_0$ with a constant speed of V_0 electron volts.

Appendix A

Proof of Convergence of Series-Expansion (23a)

It is to be shown that

$$S_{(t)} = \sum_{n=1}^{\infty}(-1)^n r_{(t)}^{(n-1)}\cdot Q_n(t) \quad (56)$$

is convergent. Let k_{max} be the greatest absolute value of the coefficient of deflection existing in the tube. Then, assuming that k takes the value of k_{max} everywhere along the axis, we can establish the following upper limit for Q_n

$$|Q_n| \leq 1/\mu\cdot K_{max}\cdot t^n/n!. \quad (57)$$

We assume $r(t)$ to be analytic for $t>0$. If $r(t)$ is analytic for $t=0$, it must also be analytic within a circle of radius ϵ around $t=0$. Hence, for any $0\leqq t\leqq T$, $r(t)$ is analytic within a circle of radius $(t+\epsilon)$ and center t. For such functions, Cauchy's inequality[19] holds:

$$|r^{(n-1)}| < M\cdot(n-1)!/(t+\epsilon)^{n-1} \quad (58)$$

where M is the maximum value of $r(t)$ within that circle. From (57) and (58):

$$S = \sum_{n=1}^{\infty}|r^{(n-1)}||Q_n| < \frac{K_{max}Mt}{\mu}\sum_{n=1}^{\infty}\frac{1}{n}\cdot\left(\frac{t}{t+\epsilon}\right)^n. \quad (59)$$

This series is convergent, because $t/(t+\epsilon)$ is a proper fraction for $0\leqq t\leqq T$. The convergence of (59) is a sufficient condition for the convergence of (56).

Appendix B

Evaluation of the Impulse Integral Q_1

Ability to determine the first integral Q_1 implies a knowledge of the second derivative of the potential and Q_1 is therefore not readily obtained from graphical representations. Its computation may be greatly simplified if an analytical function may be found, which approximates the experimental distribution closely. In the case of the electron gun, it has been found that data from the

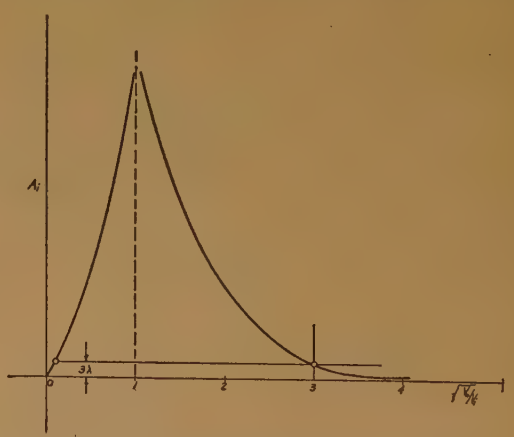


Fig. 10—Focal length of a bipotential lens as a function of electrode-voltage ratio.

electrolytic trough may be represented fairly well by the hyperbolic tangent

$$V_s = V_m[1 + \alpha\tanh(z - z_m)/\lambda] \quad (60)$$

where

$$V_m = (V_i + V_0)/2; \qquad \alpha = (V_i - V_0)/(V_i + V_0).$$

In this equation, 2λ is the subtangent at the point of symmetry as shown in Fig. 7b. Using (56), the first

[18] This neglects the initial axial electron-velocities at the cathode surface.

[19] E.T.Whittacker and G.N.Watson, "A Course of Modern Analysis," fourth edition, Cambridge University Press, London, England, 1927, page 91, §5.23.

integral Q_1, or the impulse of the lens, may immediately be written

$$Q_1 = \sqrt{V_m}/\alpha \cdot \lambda[(\gamma_z^{1/2} - (1/3)\gamma_z^{3/2}) - (\gamma_0^{1/2} - (1/3)\gamma_z^{3/2})] \quad (61)$$

where

$$\gamma_z = V_z/V_m.$$

This has been plotted in Fig. 7c.

In this connection, it may be shown that symmetric fields, as expressed by (60) exhibit positive lens action under all circumstances. Equation (61) may be brought into the following form between the limits $V=V_0$ and $V=V_i$:

$$Q_{1i} = (1/3\lambda)(\sqrt{V_i} - \sqrt{V_0})^2/(\sqrt{V_i} + \sqrt{V_0}). \quad (61a)$$

This is seen to be always positive regardless of whether the gun voltage is lower or higher than the final electrode voltage. For the focal length, we find from (39)

$$f_i = 3\lambda \frac{1 + \sqrt{V_0/V_i}}{(1 - \sqrt{V_0/V_i})^2}. \quad (39a)$$

This, too, is found to be positive for both accelerating and decelerating systems. It is plotted in Fig. 10. Apparently, $f=3\lambda$ is about the shortest focal length which may be obtained with either very small gun voltages or quite large ones $V_0/V_i=1/10$, and $V_0/V_i=10$. This shortest focal length is about twice the gun diameter.

Acknowledgment

The author wishes to express his gratitude to Mr. Arthur S. Gincberg, Physics Department, Purdue University, who supplied the proof of convergence of the infinite series of functions used.

The assistance of Mr. Donald J. Tendam, of the same department was a great help in the preparation of the manuscript, and is gratefully acknowledged.

The following statement is a revision of the material appearing on pages 175 and 176 of the March, 1944, issue of the PROCEEDINGS. The attention of the reader is accordingly directed to the corresponding changes.

The Editor

Standard-Frequency Broadcast Service of National Bureau of Standards*

THIS service comprises the broadcasting of standard frequencies and standard time intervals from the Bureau's radio station WWV near Washington, D. C. It is continuous at all times day and night, from 10-kilowatt radio transmitters, except on 2500 kilocycles per second where 1 kilowatt is used. The services include: (1) standard radio frequencies, (2) standard time intervals accurately synchronized with basic time signals, (3) standard audio frequencies, (4) standard musical pitch, 440 cycles per second, corresponding to A above middle C.

The standard-frequency broadcast service makes widely available the national standard of frequency, which is of value in scientific and other measurements requiring an accurate frequency. Any desired frequency may be measured in terms of any one of the standard frequencies, either audio or radio. This may be done by the aid of harmonics and beats, with one or more auxiliary oscillators.

At least three radio carrier frequencies are on the air at all times, to insure reliable coverage of the United States and other parts of the world. The radio frequencies are:

2.5 megacycles (=2500 kilocycles=2,500,000 cycles) per second, broadcast from 7:00 P.M. to 9:00 A.M., Eastern War Time (2300 to 1300 Greenwich Mean Time).

5 megacycles (=5000 kilocycles=5,000,000 cycles) per second, broadcast continuously day and night.

10 megacycles (=10,000 kilocycles=10,000,000 cycles) per second, broadcast continuously day and night.

15 megacycles (=15,000 kilocycles=15,000,000 cycles) per second, broadcast from 7:00 A.M. to 7:00 P.M. Eastern War Time (1100 to 2300 Greenwich Mean Time).

Two standard audio frequencies, 440 cycles per second and 4000 cycles per second, are broadcast on the radio carrier frequencies. Both are broadcast continuously on 10 and 15 megacycles. Both are on the 5 megacycles in the daytime, but only the 440 is on the 5 megacycles from 7:00 P.M. to 7:00 A.M., Eastern War Time. Only the 440 is on the 2.5 megacycles.

The 440 cycles per second is the standard musical pitch, A above middle C; the 4000 cycles per second is a useful standard audio frequency for laboratory measurements.

In addition there is on all carrier frequencies a pulse of 0.005 second duration which occurs at intervals of precisely 1 second. The pulse consists of 5 cycles, each

* Decimal classification: R355. Original manuscript received by the Institute, June 19, 1944.

of 0.001 second duration, and is heard as a faint tick when listening to the broadcast; it provides a useful standard of time interval, for purposes of physical measurements, and may be used as an accurate time signal. On the 59th second of every minute the pulse is omitted.

The audio frequencies are interrupted precisely on the hour and each 5 minutes thereafter; after an interval of precisely 1 minute they are resumed. This 1-minute interval is provided in order to give the station announcement and to afford an interval for the checking of radio-frequency measurements free from the presence of the audio frequencies. The announcement is the station call letters (WWV) in telegraphic code (dots and dashes), except at the hour and half hour when a detailed announcement is given by voice.

The accuracy of all the frequencies, radio and audio, as transmitted, is better than a part in 10,000,000. Transmission effects in the medium (Doppler effect, etc.) may result at times in slight fluctuations in the audio frequencies as received; the average frequency received is however as accurate as that transmitted. The time interval marked by the pulse every second is accurate to 0.000.01 second. The 1-minute, 4-minute, and 5-minute intervals, synchronized with the seconds pulses and marked by the beginning or ending of the periods when the audio frequencies are off, are accurate to a part in 10,000,000.

The beginnings of the periods when the audio frequencies are off are so synchronized with the basic time service of the United States Naval Observatory that they mark accurately the hour and the successive 5-minute periods.

Of the radio frequencies on the air at a given time, the lowest provides service to short distances, and the highest to great distances. Reliable reception is in general possible at all times throughout the United States and the North Atlantic Ocean, and fair reception throughout the world.

Information on how to receive and utilize the service is given in the Bureau's Letter Circular, "Methods of using standard frequencies broadcast by radio," obtainable on request. The Bureau welcomes reports of difficulties, methods of use, or special applications of the service. Correspondence should be addressed to the National Bureau of Standards, Washington, D. C.

I.R.E. People

Dr. Armstrong Receives Certificate from General Ingles

Certificate of Appreciation to Dr. Armstrong

Dr. Edwin H. Armstrong has been presented with the Chief Signal Officer's Certificate of Appreciation by Major General H. C. Ingles, Chief Signal Officer, at a recent ceremony in the Pentagon Building at Arlington, Virginia.

In handing the first Certificate to Dr. Armstrong in recognition of "loyal and patriotic services" during two wars, General Ingles said in part:

"Those of us who have been associated with you through the years know how unsparingly you have contributed your talents and your time to the development of Signal Corps equipment which is now proving its superiority on every front. We recall that you perfected the superheterodyne receiver during the first World War and your more recent waiver of royalties on your frequency-modulation patents is still fresh in our memories. In addition, you have undertaken vital contract development work for the Signal Corps and given generously of your knowledge and advice in the conduct of many experiments."

The Certificate was designed as a testimonial to individuals and companies who have performed notable services beyond the normal requirements of duty but who are not under the direct control of the War Department and are therefore not eligible for the Secretary of War's civilian award or the Army-Navy "E."

THE INSTITUTE OF RADIO ENGINEERS

INCORPORATED

SECTION MEETINGS

ATLANTA	CHICAGO	CLEVELAND	DETROIT	LOS ANGELES
September 15	September 15	September 28	September 15	September 19
NEW YORK	**PHILADELPHIA**	**PITTSBURGH**	**PORTLAND**	**WASHINGTON**
September 6	October 5	October 9	August 14	October 9

SECTIONS

ATLANTA—Chairman, Walter Van Nostrand; Secretary, Ivan Miles, 554—14 St., N. W., Atlanta, Ga.

BALTIMORE—Chairman, G. J. Gross; Secretary, A. D. Williams, Bendix Radio Corp., E. Joppa Rd., Towson, Md.

BOSTON—Chairman, R. F. Field; Secretary, Corwin Crosby, 16 Chauncy St., Cambridge, Mass.

BUENOS AIRES—Chairman, L. C. Simpson; Secretary, I. C. Grant, Venezuela 613, Buenos Aires, Argentina.

BUFFALO-NIAGARA—Chairman, A. J. Dybowski; Secretary, H. G. Korts, 51 Kinsey Ave., Kenmore, N. Y.

CHICAGO—Chairman, W. O. Swinyard; Secretary, A. W. Graf, 4 Midway Ct., Hammond, Ind.

CINCINNATI—Chairman, J. L. Hollis; Secretary, R. S. Butts, Box 1403, Cincinnati 2, Ohio.

CLEVELAND—Chairman, A. S. Nace; Secretary, Lester L. Stoffel, 1095 Kenneth Dr., Lakewood, Ohio

CONNECTICUT VALLEY—Chairman, R. E. Moe; Secretary, L. A. Reilly, 989 Roosevelt Ave.; Springfield, Mass.

DALLAS-FORT WORTH—Chairman, D. J. Tucker; Secretary, P. C. Barnes, WFAA-WBAP, Grapevine, Texas.

DAYTON— Chairman, I. H. Gerks; Secretary, Joseph General, 1319 Superior Ave., Dayton, 7, Ohio.

DETROIT—Chairman, R. A. Powers; Secretary, R. R. Barnes, 1411 Harvard Ave., Berkley, Mich.

EMPORIUM—Chairman, H. D. Johnson; Secretary, A. Dolnick, Sylvania Electric Products, Inc., Emporium, Pa.

INDIANAPOLIS—Chairman, A. N. Curtiss; Secretary, E. E. Alden, WIRE, Indianapolis, Ind.

KANSAS CITY—Chairman, A. P. Stuhrman; Secretary, R. N. White, 4800 Jefferson St., Kansas City, Mo.

LOS ANGELES—Chairman, L. W. Howard; Secretary, Frederick Ireland, 1000 N. Seward St., Hollywood, 38, Calif.

MONTREAL—Chairman, F. S. Howes; Secretary, J. A. Campbell, Northern Electric Co., Ltd., 1261 Shearer St. Montreal, Que., Canada.

NEW YORK—Chairman, Lloyd Espenschied; Secretary, J. E. Shepherd, 111 Courtenay Rd., Hempstead, L. I., N. Y.

PHILADELPHIA—Chairman, T. A. Smith; Secretary, S. Gubin, RCA Victor Division, Radio Corporation of America Bldg. 8–10, Camden, N. J.

PITTSBURGH—Chairman, T. C. Kenny; Secretary, R. K. Crooks, Box 2038, Pittsburgh 30, Pa.

PORTLAND—Chairman, W. A. Cutting; Secretary, W. E. Richardson, 5960 S.W. Brugger, Portland, Ore.

ROCHESTER—Chairman, G. R. Town; Secretary, Harold Goldberg, Stromberg-Carlson Co., Rochester 9, N. Y.

ST. LOUIS—Chairman, N. B. Fowler; Secretary, C. H. Meyer, KFUO, 801 DeMun Ave., St. Louis, Mo.

SAN FRANCISCO—Chairman, W. G. Wagener; Secretary, R. V. Howard, Associated Broadcasters, Inc., Mark Hopkins Hotel, San Francisco 6, Calif.

SEATTLE—Chairman, F. B. Mossman; Secretary, E. H. Smith, Apt. K, 1620—14 Ave., Seattle, 22, Wash.

TORONTO—Chairman, E. O. Swan; Secretary, Alexander Bow, Wire Products, Ltd., 2738 Dundas St., W., Toronto, Ont., Canada.

TWIN CITIES—Chairman, E. S. Heiser; Secretary, B. R. Hilker, KSTP, St. Paul Hotel, St. Paul, Minn.

WASHINGTON—Chairman, J. D. Wallace; Secretary, F. W. Albertson, c/o Dow and Lohnes, Munsey Bldg., Washington, D. C.

WILLIAMSPORT—Chairman, H. E. Smithgall, Jr.; Secretary, K. E. Carl, Williamsport Technical Institute, Williamsport, Pa.

Institute News and Radio Notes

Constitutional Amendment Section

Institute Dues are Low

The dues in the Institute are far below what they should be, judging by comparable dues of other engineering societies. An average for ten such societies is $20, $18.80, and $12+ for grades corresponding to our Fellow, Senior Member, and Member-Associate. Our dues are $10, $10, and $6 and the Board proposes $15, $15, $10–$7, which is still very moderate.

Plans

The Board will, if the dues are increased, plan as far as possible to restore its activities to the prewar standard. The office staff must be increased, and larger quarters must be secured. The Board will make plans so that the flood of technical papers expected after the war can be promptly handled. The technical developments during the past three years are astounding in number and scope. The membership will want to read about them just as quickly as the articles can be put into print. They will chafe if there is delay due to a rationing out of Proceedings pages as has happened in past times. It will be worth money to the membership to secure prompt publication. It will cost money to do it.

The Board has received complaints that certain subjects are seldom if ever treated in the Proceedings. The Board would like to cover the entire field each year. Additional editorial and papers-procurement staff are necessary if this is to be accomplished.

Many sections ask for larger rebates. This can be done if the dues are increased sufficiently.

The Board wishes the Institute to be able to do other things related to the activities of the Radio Technical Planning Board which it helped to found, and to which it contributes for its operation.

The Board has received requests for other kinds of services. The Board would like to give these services to the members. At present it is not possible to do so.

Summary

Prior to the war, the Institute was having difficulty staying within its budget. Wage levels have since increased. Paper, printing, and rents have risen. Increased advertising has been a much-needed aid, but cannot primarily be depended upon for postwar operation. The Institute dues are much lower than in other societies. A flood of technical papers is expected after the war. Increased services to members should be given. The Board believes an increase in dues will place the Institute in a safer position not only to carry on at its present standard, but to do more for its members in the future.

A Message from the President

The Constitution of the Institute provides a means by which a group of members may submit a petition to amend it. Recently the President received such a petition which he submitted to the Board of Directors, and which will be mailed with a letter ballot to each voting member after an opportunity has been given for discussion in the Proceedings and at Section Meetings. This latter arrangement is in accordance with a suggestion the President has made on previous occasions that such opportunity be given in the case of all Amendments submitted to the membership.

Since no letter accompanied the petition the petitioners' reasons for offering the proposed amendment are not known. It reads as follows:

> "No person shall be eligible for appointment by the Board as Director, Secretary, Treasurer, or Editor after having accepted five such appointments to any or several of these offices."

Since this proposal does not now apply to the person who is your President he feels it to be his obligation to present his views regarding it to the Members of the Institute, which are as follows:

1. The President and Vice-President serve the Institute by giving thought to the broader policy matters, but it is the Administrative Officers, namely, the Secretary, Treasurer, and Editor who are responsible for the day-by-day work. Since the Board of Directors is authorized by the Constitution to manage the affairs of the Institute, it must have the authority to appoint the Administrating Officers of its selection. The proposed Amendment would greatly limit the Board in carrying out its functions.
2. Since the Constitution provides that the Secretary, Treasurer, and Editor must be Members of the Board, they cannot avoid holding two appointed offices at the same time; therefore, under the present proposal they would have to be replaced by inexperienced men at the end of two years. This is an unbusinesslike and impracticable arrangement, as under it the Institute would be operated by officers who would not have an opportunity to serve long enough to become familiar with their duties, an unthinkable situation. Most technical and scientific societies continue their officers over long periods of time.

3. The Constitution provides that three Directors are elected every year for three year terms, in order to provide overlap and continuity of experience and policy. The proposed Amendment does not provide for any method whereby continuity of experience and efficiency can be secured for the important jobs of Secretary, Treasurer, and Editor.
4. The field from which the Board can secure suitable candidates for these posts is very limited, and the factors that so limit it are beyond the control of the Institute. The qualifications for these positions must be high and stringent. The duties involved are numerous and complicated. The Officers are called upon to attend at least ten Board Meetings and twelve Executive Committee Meetings or a total of twenty-two a year not to speak of other urgent meetings and calls upon their time. The candidates must be close enough to New York so that they are easily accessible to Headquarters. The qualifications require a willingness to serve and also to devote, without compensation, the very considerable amount of time involved. There are indeed few men having these qualifications and of these only a few whose services can be obtained. In my opinion the limitations imposed by the proposed Amendment would, because of this reason alone, greatly lower the standards of management of your Institute in the course of a short time.

COMMUNICATION

318 Monmouth Avenue,
Town of Mount Royal,
P.Q. June 16, 1944.

The Editor,
Proceedings of the I.R.E.,
New York, N. Y.

DEAR SIR,

I am a little surprised that Lt. Comdr. Van Dyck should have adopted the academic type of defence in his letter of 17th May regarding the Montreal petition. In doing so he has revealed the issue in a very fortunate manner. From his definitions it is sufficiently clear that the word "Associate" may properly be applied to those with inherent qualifications, whereas "Affiliate" signifies a membership which is granted as an act of goodwill towards one who is wanted in the Institute but who cannot claim membership as a natural right.

So long as this is an institute of radio engineers, and not a fraternal order of those interested in radio, it would seem that Lt. Comdr. Van Dyck's recourse to the dictionary has lent ample support to the stand taken by the Montreal petitioners. I, for one, like his definitions and hope he will abide by them. The real issue, of course, is whether the Institute of Radio Engineers is to be what in turn its name implies.

Yours faithfully,
L. T. BIRD

Board of Directors

June 7 Meeting: At the regular meeting of the Board of Directors, which was held on June 7, 1944, the following were present: H. M. Turner, president; R. A. Hackbusch, vice-president; S. L. Bailey, Alfred N. Goldsmith, editor; R. F. Guy, R. A. Heising, treasurer; F. B. Llewellyn, Haraden Pratt, secretary; B. J. Thompson, L. P. Wheeler, W. C. White, and W. B. Cowilich, assistant secretary.

Membership: The following applications for membership, which were recommended by the Executive Committee, were approved: for transfer to Senior Member grade, R. T. Gabler, F. L. Hopper, J. L. Potter, J. D. Schantz, L. E. Thompson, and W. L. Webb; for admission to Senior Member grade, Pierre Mertz and W. W. Salisbury; for transfer to Member grade, V. H. Campbell, A. P. Chesney, Albert Dolnick, W. E. Dulin, W. C. Freeman, Jr., E. M. Johnson, W. D. Johnson, M. B. Kline, W. P. Mueller, J. G. Rountree, M. O. Schilling, Herbert Sherman, H. E. Smithgall, Jr., and W. H. Warren; for admission to Member grade, A. G. Cooley, D. W. Hunt, V. C. Rideout, F. B. Schramm, and T. D. Talmage; Associate grade, 174; and Student grade, 77.

Executive Committee Actions: The actions of the Executive Committee at its May 2, 1944 meeting were unanimously ratified.

Committees: On recommendation of the Executive Committee, the following members were appointed to the particular committees indicated below:

INVESTMENTS
Fulton Cutting
PAPERS PROCUREMENT
ELECTRON TUBES
J. R. Nelson, *Chairman*
INSTRUMENTATION
Howard Tyzzer, *Chairman*
RADIO BROADCASTING
R. E. Coram — G. W. Lang
Henry Grossman — P. F. Robinson
R. E. Shelby
TIMERS AND TECHNICAL CONTROLS
J. M. Cage — J. P. Taylor
V. M. Sherman — V. E. Trouant
P. B. Weiss

The Board expressed its appreciation of the efforts of Mr. Dorman Israel, in his capacity as Chairman of the Papers Procurement Committee, and its gratification with the excellent results from the work done, and unanimously authorized Editor Goldsmith to convey these expressions to Mr. Israel.

Technical

CIRCUITS
J. M. Miller
ELECTRONICS
E. C. Homer — J. R. Steen
RADIO WAVE PROPAGATION
R. L. Smith-Rose
TELEVISION
George Fyler — H. T. Lyman
SYMBOLS
E. W. Shafer, *Chairman*

Deaths

Stuart Ballantine: President Turner reported that Stuart Ballantine, who was President of the Institute in 1935, died on May 7, 1944. Flowers and a letter of condolence were sent by the Institute to Mrs. Ballantine and her reply was read at the meeting.

Miss Agnes Irene Dunne: The Board of Directors sent flowers to the late Miss Dunne, who died on May 14, 1944, and a letter of condolence to the family. Miss Dunne was an employee of the Institute since its inception.

Certificate of Incorporation: Treasurer Heising stated that copies of the recently amended charter of the Institute were mailed to the Board members on May 31, 1944.

WPB Paper Limitation Order L-244: Another written appeal from this order had been mailed on May 25, 1944, to the War Production Board, in a further attempt to obtain the additional paper allotment needed for increasing the substance weight of the paper stock, the number of pages, and the number of copies for new members and subscribers.

Collective Bargaining: The committee report, dated June 5, 1944, was reviewed by Chairman White. The recommendation of

the committee, in the revised form quoted below, was unanimously approved.

"That, a statement be prepared, with the aid of counsel, on the various aspects of collective bargaining for radio engineers. After review by the Committee on Professional Recognition, it is to be submitted for approval by the Board for distribution to the membership either by publication in the PROCEEDINGS or in the form of a pamphlet available to members on request."

Canadian Membership: Vice-President Hackbusch reviewed the developments in the Canadian situation relating to collective bargaining for engineers, and also referred to the Canadian Wartime Labour Relations Regulations known as Order-in-Council P.C. 1003 on the subject of collective bargaining.

Particular attention was called to the steps being taken by the Toronto and Montreal Sections to circularize their members on the question of professional recognition and collective bargaining.

Nominations: The following nominations were approved by the Board of Directors for the 1945 elective offices:

PRESIDENT
W. L. Everitt
VICE-PRESIDENT
H. J. van Bijl
DIRECTORS
1945–1948

S. L. Bailey	F. M. Ryan
E. F. Carter	B. E. Shackelford
Keith Henney	W. O. Swinyard

Constitution and Laws

Constitution: Attention was called to the fact that the Constitutional amendments, which had been approved by the voting membership in 1943, were adopted by the Board of Directors at its meeting on September 8, 1943, and not on October 8, 1943 as incorrectly shown in the current copies of the Constitution.

It was unanimously approved to use September 8, 1943, as the date of adoption of the 1943 Constitutional amendments, when reprinting the Constitution and any other forms containing a reference to the particular date.

Constitutional Amendments: The following proposed amendment was adopted to be voted on together with the proposed "Montreal amendment":

ARTICLE IV

Section 1—The proposed amendment, to be worded as quoted and on the ballot to be accompanied by the dual explanation stated in parentheses, was unanimously approved.

Under the heading "Annual Dues":

"Delete '$10' after word 'Fellow' and substitute '$15.'

"Delete '$10' after words 'Senior Member' and substitute '$15.'

"Delete '$6' after word 'Member' and substitute '$10.'

"Delete '$6' after word 'Associate' and substitute '$7,' for the first five years of membership as (Affiliate and prior membership as)* Associate; thereafter $10 beginning the January first following. The clause changing the dues from $7 to $10 after five years of membership as (Affiliate and prior membership as)* Associate shall not take effect until January 1, 1946.'

"* Note—Parentheses will be eliminated if the 'Montreal amendments' are adopted, while the words within the parentheses will be deleted if such amendments fail."

(The wording of this amendment, excluding parenthetical parts and note, is drawn up to apply to the present Constitution as it now stands, and affects only the figures and words mentioned. The "Montreal amendments" also on this ballot, if approved, will change other words independently of this amendment. Wordings which are contingent upon the adoption of the "Montreal amendments" are in parentheses and are noted.)

ARTICLE II

Section 1A—The wording of this proposed amendment, previously approved, was revised to read as follows:

"Article II, Section 1A—At the time of the adoption of the seventh amendment, all Senior Members shall become Members, all Members shall become Associates, and all Associates shall become Affiliates. Associates of record for three years shall be sent a notice of such change and shall be given an opportunity good for six months to be restored to continuous Associate membership by showing that they are or have been engineers, or scientists, or teachers in radio or allied fields."

The following amendments were approved by the Board:

ARTICLE I

Section 2—Insert after the first word "be" the following words "scientific, literary, and educational. Its aims shall include" making it read "Sec. 2—Its objects shall be scientific, literary, and educational. Its aims shall include the advancement of the theory and practice of radio, etc."

ARTICLE II

Section 1e—The proposed amendment of Article II, Section 1e, indicated below, was unanimously approved.

Delete in Article II, Section 1e, everything after the word "Student" and substitute "who may participate in meetings, wear the badge of the Institute, and receive publications designated by the Board of Directors, but who shall have no other rights and privileges."

ARTICLE VII

Section 1—This additional proposed amendment of Article VII, Section 1, was unanimously approved. Insert in the second paragraph, second sentence, between the words "before" and "August," the words "twelve o'clock noon on the last weekday prior to" making it read "For acceptance, a letter of petition must reach the executive office before twelve o'clock noon on the last weekday prior to August fifteenth of any year, etc."

In the fifth paragraph, fourth sentence, insert between the words "office" and "prior" the words "before twelve o'clock noon on the last weekday" making it read "Only ballots arriving at the executive office before twelve o'clock noon on the last weekday prior to October twenty-fifth shall be counted."

ARTICLE IX

Delete title and both Sections 1 and 2. Substitute—"Sections and Other Groups."

Section 1. The Board of Directors may authorize the establishment of Sections and other groups of members for the purpose of promoting the interests of the Institute. The Board of Directors may, at its discretion, terminate the existence of any such group."

ARTICLE X

Section 2—The wording of this amendment, also previously approved, was revised as follows:

After the first sentence, insert this wording: "The ballots, after marking, shall be placed in plain sealed envelopes, enclosed within mailing envelopes marked 'ballot' and bearing the member's written signature. Only ballots within signed outer envelopes shall be counted. No votes by proxy shall be counted. Only ballots arriving at the executive office prior to the stated time limit shall be counted."

Constitutional-Amendment Petition: President Turner read an undated petition, which had been addressed to him, with the proposal that Article VII of the Constitution be amended with the addition of the following wording:

"No person shall be eligible for appointment by the Board as Director, Secretary, Treasurer, or Editor after having accepted five such appointments to any or several of these offices."

The names of the 54 signers were read by the Assistant Secretary and it was noted that 48 were voting members in good standing and six nonvoting Associates.

The petition, complying with the Constitution insofar as the required number of signatures of voting members is concerned, was referred to Chairman Heising of the Constitution and Laws Committee, for submission to legal counsel with regard to compliance with the amendment Section of the Constitution of the Institute.

President Turner requested the Assistant Secretary to send a letter of acknowledgment to each qualified signer of the petition, and also to ask for expressions of the reasons prompting the proposal, which would be used to enlighten the membership. It was further requested that such expressions should reach the Institute office by July 20, 1944, if they are to be published in the September issue of the PROCEEDINGS.

T. L. Eckersley: The quoted citation, for the Fellowship awarded to Mr. Eckersley at the last meeting, was unanimously approved:

"For his outstanding contributions to the theory and practice of radio-wave-propagation research. Both his approach to the problem from the standpoint of practical communications and his invention of mathematical tools useful in the computation of radiated fields are achievements of lasting value acclaimed by the whole radio world, and form a monument of which he may be justly proud."

Radio Technical Planning Board: Secretary Pratt, as the Institute Representative on the RTPB, reviewed his report accompanied by copies of letters to RTPB Chairman Baker. In that correspondence, Secretary

Pratt raised procedural objections to the steps taken by Chairman Baker in bringing about changes in the two matters, listed below, and explained the reasons for not voting on the matters involved:

Name and Scope of RTPB Panel 5
RTPB Publicity Policy

It was further reported that the mentioned subjects would be discussed at the next RTPB meeting, which is expected to be held in the near future.

Standardization

The following matters were given consideration from the standpoint of expanding radio-engineering standardization in the postwar period:

Radio Technical Planning Board: The quoted resolution, which was read and discussed, was referred to the Executive Committee for recommendations to be available, if possible, for the September meeting of the Board:

"Resolved, That it is the sense of the Board of Directors that the Radio Technical Planning Board has certain long-term constructive aspects in the field of engineering standardization, particularly along system lines, and

"That such activities of the RTPB are of particular interest to the I.R.E., and

"That the Institute should, from time to time, support, sponsor, and appropriately contribute to the guidance of such activities, and

"That the Institute should be prepared to act, at the request of the RTPB, as an analytic and validating group between the RTPB and the Sectional Committee on Radio of the ASA, as well as the International Standards Association or an equivalent international standardizing body, and

"That the President of the Institute is empowered and requested to study and report such activities to the Board and to recommend that such steps as he deems appropriate shall be taken in relation therewith."

ASA Special Conference on Planning: Dr. Llewellyn stated that this conference, held on May 5, 1944, was devoted to a discussion of standardization after the war.

1945 Winter Technical Meeting: President Turner reported the action of the Executive Committee in approving the listed dates and program and the stated meeting-name, which had been proposed by Dr. Austin Bailey, chairman of that meeting: January 24 to 27, 1945, inclusive.

Executive Committee

June 6, 1944, Meeting: The following members were present at the June 6, 1944, meeting of the Executive Committee: H. M. Turner, president; E. F. Carter, Alfred N. Goldsmith, editor; R. A. Heising, treasurer; F. B. Llewellyn, Haraden Pratt, secretary; and W. B. Cowilich, assistant secretary.

Amendment of Certificate of Incorporation: Treasurer Heising stated that a certified copy of the Institute's revised charter, filed on May 2, 1944, with the New York Department of State under the official title quoted, had been received from General Counsel Zeamans, and that copies of it had been mailed to the Board members. "Certificate of Change of Purposes and Powers and Provisions of Certificate of Incorporation, Change in Number of Directors, Change in Time of Holding Annual Meeting of The Institute of Radio Engineers, Incorporated."

ASA Special Conference on Planning: Dr. Llewellyn reported on the particular conference, held on May 5, 1944, minutes of which had been received by the Institute.

President Turner, who attended the meeting, described the views which he expressed there and which are summarized in this extract from the mentioned minutes: "Professor Turner of the Institute of Radio Engineers said his group was especially interested in international standardization. The ASA should survey the wartime standards to determine which should be discarded; continue the work of co-ordination of standards with the Canadian, British, and South American groups; strive toward world standards through the International Electrotechnical Commission; and co-ordinate the work of the Radio Technical Planning Board with the ASA." The subject was discussed and referred to the Board for further consideration.

Proceedings: As a result of the reduced number of extra copies printed, necessitated by the WPB paper-limitation regulations, the March and April, 1944, issues of the PROCEEDINGS are now entirely out of stock.

National Electronics Conference

In keeping with its objective to serve as "a national forum on electronics and its engineering applications" two addresses keynoting the National Electronics Conference will be delivered. These are "Electronic Research Opens New Frontiers," by Ralph Beal and "Electronics In Industry," by W. C. White.

Technical papers on a wide range of industrial, medical, measurement, and communication topics will be presented at the conference to be held at the Medinah Club of Chicago on October 5 to 7, 1944. As Chairman of the Program Committee, Professor A. B. Bronwell, Northwestern University, Evanston, Illinois, has prepared a program of broad interest covering the entire electronic field. Subject to minor changes and several additions already planned, the technical program will include presentation of the following papers:

"Color and Ultra-High Frequency Television," by P. C. Goldmark, Columbia Broadcasting System

"A Lighthouse Tube; A Pioneer Ultra-High-Frequency Development," by E. D. McArthur and E. F. Peterson, General Electric Company

"Audible Audio Distortion," by H. H. Scott, General Radio Company

"Principles of Klystron Amplifiers," by Robert Haxby, Sperry Gyroscope Company

"Radio Relay Systems," by C. W. Hansell, RCA Laboratories

"Broad-Band Carrier and Coaxial-Cable Networks," by F. A. Cowan, American Telephone and Telegraph Company

"Development of Electronic Tubes," by I. E. Mouromtseff, Westinghouse Electric and Manufacturing Company

"Theorem of Lorentz and Its Importance for Problems of Electrons in Magnetic Fields," by Leon Brillouin, Columbia University

"Theory of Microwave Oscillation Generators Using Velocity-Modulated Electron Beams," by E. U. Condon, Westinghouse Electric and Manufacturing Company

"The Supersonic Eccoscope, An Instrument for Inspecting the Interior of Metal Parts by Means of Sound Waves," by F. A. Firestone, University of Michigan

"Electronic Mechanisms in the Process Plant and Industrial Laboratory," by T. A. Cohen, Wheelco Instrument Company

Dynamic Strain Gages," by C. A. Dohrenwend, Armour Research Foundation

"High-Frequency Induction Heating of Metals," by C. J. Madsen and R. M. Baker, Westinghouse Electric and Manufacturing Company

"New Methods and Techniques in High-Frequency Heating," by Eugene Mittelmann, Illinois Tool Works

"Electronic Power Converters," by E. F. W. Alexanderson, General Electric Company

"Electronic Measurements of Nonelectrical Quantities in Industrial Processes," by H. D. Middell, General Electric Company

"The Mass Spectrometer and its Practical Applications," by J. Hipple, Westinghouse Electric and Manufacturing Company

"Design Factors in the Application of Relays to Electronic Circuits," by R. H. Herrick, Automatic Electric Company

A discussion forum on educational problems in the electronic field will be held on Saturday, October 7 with representatives of engineering colleges.

Announcement of the final program, together with details concerning the National Electronics Conference will be made about September 1, when registration for the conference will be accepted by Professor P. G. Andres, Chairman of the Arrangements Committee, who may be addressed at the Illinois Institute of Technology, 3300 Federal Street, Chicago, Illinois.

Correspondence

Father-and-Son Night for Sections

The following communication was originally addressed to Dr. B. E. Shackleford, chairman of the Membership Committee of the New York Section and a member of its Executive Committee. Certain favorable reactions to the proposed plan prompt its presentation in these columns. It is suggested that the Section officers consider this or similar plans and communicate their viewpoints to Mr. W. B. Cowilich, Assistant Secretary of the Institute of Radio Engineers, who will co-ordinate these opinions and present them to the Executive Committee and Board of Directors for their information and guidance.

The Editor

Dr. B. E. Shackleford
Radio Corporation of America
New York 20, New York

Dear Dr. Shackleford:

During the past few years the Institute of Radio Engineers has experienced a most gratifying growth in membership. Part of this has been the result of the untiring efforts of the Membership Committee, part to our many good friends in the I.R.E., who, from time to time, solicit people in their organizations, and part is attributable to the War Effort, since thousands of men, who might have been otherwise engaged, are now working on radio-and-electronic problems. In order that the splendid progress we have made may continue, it is necessary that we do not relax our efforts, but go even further in establishing a solid foundation for future memberships.

Several years ago the Institute recognized the importance of interesting the college and university students in our organization. This had a twofold purpose; altruistically, to assist the student and to give him a broader vision of the radio engineering field; practically, to secure members who eventually would be among the leaders in their profession. This plan has worked out exceedingly well and has been to the mutual benefit of both the student and the I.R.E.

It would appear that even more attention might be given to the younger group of men who are now in the Institute and to those even younger boys who are not eligible for membership. Many of the more mature members of the I.R.E. have growing sons, some of whom are intensely interested in radio.

To foster this growing interest, the Sections might consider the idea of a Father-and-Son night once a year, or more as they see fit. Since the I.R.E. was founded on May 13, this day, or a day near that date, might well serve as an appropriate night; thus, too, Founders' Day could come to have more importance in the active life of the Institute.

Such a meeting might well take the form of a dinner followed by a semitechnical program. Several movies are available—"Electronics at Work" and "Crystals Go to War," for example. A display of old and new radio equipment with someone to explain it would be interesting, particularly if the boys were permitted to handle the material. A trip of a transmitting station or broadcast studio might also serve. It is reasonable to presume that the Fathers would be pleased by the opportunity to have their sons meet the sons of their friends in the I.R.E. and the boys undoubtedly would enjoy it.

If the Institute wished to interest this younger group more extensively, a junior section might be organized with certificates issued to qualifying members and a name, such as "Sparks" given it. Meetings could be opened to all boys interested and a committee made responsible to lead and assist the boys. The chairman of the committee might be an adult Section member and his committee men Student members in the community. Thus, our Student members would form a link between the fathers and their sons and would have the opportunity of serving the Institute in an executive capacity. An annual prize might be offered to the boy writing the best paper on a selected subject. If enough interest were shown, a small publication, containing elementary papers on radio and electronics, might be published.

Obviously, when the boys from such groups as those proposed above, go to college, the matter of Student membership will be taken for granted. At the conclusion of their academic life, membership in the I.R.E. will be but a simple transfer. The Institute thereby would practically handpick its future members and bring them up in the best interests of the engineering fraternity. The high standing of the Institute would be raised even higher by this procedure, for our Associate members would be the cream of the crop and would be a group which for many years had been under the influence of the I.R.E.

Helen M. Stote
Associate Editor

Correspondence on both technical and nontechnical subjects from readers of the PROCEEDINGS OF THE I.R.E. is invited, subject to the following conditions: All rights are reserved by the Institute. Statements in letters are expressly understood to be individual opinion of writer, and endorsement or recognition by the I.R.E. is not implied by publication. All letters are to be submitted as typewritten, double-spaced, original copies. Any illustrations are to be submitted as inked drawings. Captions are to be supplied for all illustrations.

Books

Industrial Electronic Control, by W. D. Cockrell

Published (1944) by McGraw-Hill Book Company, 330 West 42nd St., New York 18, N. Y. 242 pages+5-page index+xii pages. 175 illustrations. 8½×5½ inches. Price, $2.50.

This book appears to be intended for use primarily by technical men having little or no previous background in electronics; for example, mechanical, civil, chemical, or electric power engineers, who find themselves engaged in work in which a knowledge of the applications of electronic devices in the manufacturing industries is important. The book is descriptive rather than highly analytic in nature, and is evidently intended to impart a quick rather than a thorough knowledge. There are many very good and well-prepared circuit diagrams, also diagrams showing instantaneous variations of voltage and current in electronic circuits. No mathematical analyses of any kind are attempted, the author depending entirely on qualitative descriptions of the various relationships involved to convey an understanding of circuit behavior. The descriptions of the sequence of circuit events in various types of industrial electronic circuits are brief but good. The book should be useful.

The treatment begins with a description of vacuum tubes, both high-vacuum and gaseous. This description presumes on the part of the reader only such knowledge as any engineer may be expected to have acquired as a part of his basic training. There follow brief descriptions of the important nonelectronic components used in electronic circuits, such as transformers, inductors, capacitors, etc., also very brief comments on measuring instruments. The simpler rectifier, amplifier, oscillator, timing, and phase-shift circuits are then discussed descriptively and very briefly. The latter part of the book contains fairly detailed qualitative analyses of the behavior of the more common photoelectric, motor-control, and welder-control circuits.

This book is a first edition, and is up to date. It obviously profits greatly from the practical experience and background of its author, who is engaged in work in the industrial electronic field with the General Electric Company. The whole treatment is practical in nature. Although it does not include thorough analytical treatments of any part of the field of electronics, it gives references to textbooks which do give such analytical treatments. Of course, no book of 242 pages can include an elementary introduction to the field, and in addition a comprehensive survey coverage of all important industrial electronic circuits. There are some important applications that are treated very sketchily if at all, one of them being that of servo controls. The most serious criticism of the book is perhaps that it is overly brief.

There are several very useful appendixes, including one describing the graphical symbolism used in the book (General Electric practice is followed for the most part), one giving color coding for resistors and capacitors, one including very useful nomographs for values of capacitive and inductance reactance, one giving time-constant curves, one stating the essential rating properties of the various polyphase rectifier circuits, and one on elementary optics. Probably one of the most useful parts of the book is this optical appendix, which gives certain very elementary but essential facts of optics as related to mirrors, lenses, and light filters.

W. G. Dow
Electrical Engineering Department
University of Michigan
Ann Arbor, Michigan

Hyper and Ultra-High Frequency Engineering, by Robert I. Sarbacher and William A. Edson

Published (1944) by John Wiley and Sons, Inc., 601 West 26th St., New York 1, N. Y. 632 pages+12-page index+xv pages. 329 figures, 5¾×8¾ inches. Price, $5.50.

The stated field of coverage of this book is "all phases of hyper-frequency engineering" and to the extent that wartime conditions permit most of the field has been treated more or less thoroughly.

There is a distinct difference in styling and lack of continuity between the first several chapters and the later ones which detracts from the book's value. Through no fault of the authors, much of the material in these later chapters is obviously dated and will require revision at an early date if the book is to avoid obsolescence. Nevertheless, these chapters present a rather good story of prewar ultra-high-frequency practice which is of real value as a text.

The provision of both problems and literature references pertinent to each chapter adds a good deal to the value and usability of the book.

The field-theory treatment in the first 12 chapters is very complete but is highly mathematical and best suited for students with more mathematical maturity and theoretical interests than the average engineering student.

On the whole, this volume contains a very large amount of valuable material and will prove to be useful although the treatment is such that the entire work probably should not be presented to one class of students.

E. D. McArthur
General Electric Company
Schenectady, N. Y.

Radio Data Charts (Third Edition), by R. T. Beatty (revised by J. McG. Sowerby)

Published (1943) by Iliffe and Sons, Ltd., Dorset House, Stamford St., London S. E. 1, England. 85 pages+40 illustrations. 8×10½ inches. Price, 7/6.

These charts are a collection of nomographs which should be very useful to any one designing receivers. With the exception of electronic devices, the data cover a large portion of the problems encountered in receiver design. A brief but clear explanation and the solution of a typical problem accompanies each nomograph, and little or no knowledge of theory is necessary in making use of the abacs.

A group of five charts covers the interrelations of inductance, capacitance, and frequency throughout the range from short waves to audio frequencies. The design of inductance coils having minimum radio-frequency resistance and inductances ranging from 0.2 to 10,000 microhenries is a simple procedure using the nomographs. Two charts are devoted to showing the effects of screening cans upon inductance and resistance.

The characteristic impedance and attenuation of transmission lines as well as the Q of quarter-wave resonant lines may be quickly determined.

A number of graphs covers the design of power transformers and chokes at the British standard frequency of 50 cycles per second rather than the American standard of 60 cycles per second.

There are also included data on direct-current resistance of wire of various alloys, parallel-tuned circuits, coupled-tuned circuits, loudspeaker dividing networks, decibels, Ohm's law, power dissipation by resistors, attenuation circuits, and wire tables. In all there are 40 nomographs.

The second edition of this collection appeared some years ago. In this third edition the data have been brought completely up to date. It is top bound and paper covered.

P. S. Carter
RCA Laboratories
Rocky Point, Long Island, New York

Practical Analysis of Ultra High Frequency Transmission Lines, Resonant Sections, Resonant Cavities, Wave Guides, by J. R. Meagher and H. J. Markley

Published (1943) by RCA Service Company, Inc., Camden, N. J. 24 pages. Many figures. 8½×11 inches. Paper cover. (An I.R.E. member may obtain a copy from the publisher without cost.)

This booklet is an excellent condensed description of the more important characteristics and uses of the devices listed in its title. It is intended mainly as a pictorial background for the technician engaged in making installations and field tests. The generous use of well-selected diagrams makes the treatment useful and suggestive to the trained engineer entering this special field.

The first half is devoted to transmission lines, standing waves, resonant sections, the use of sections of lines as impedance elements or impedance-matching transformers, the tuned section as an insulator, the line-balance converter, characteristic impedance, velocity, and the excess attenuation caused by standing waves. The slotted measuring line is described, with curves to enable its use without compulsations. The remaining space is devoted to wave guides, types of propagation, methods of impedance matching, and resonant cavities.

H. A. Wheeler
Hazeltine Electronics Corporation
Little Neck, L. I., N. Y.

Klystron Technical Manual

Published (1944) by Sperry Gyroscope Company, Inc., Manhattan Bridge Plaza, Brooklyn 1, New York. 91 pages+3-page index. 26 figures. 6¼×9¼ inches. No charge. Available on request to Sperry.

According to the foreword, "This manual is concerned with the underlying principles of velocity-modulation tubes, . . . , rather than their application. . . . The present manual is intended for use by readers who have an understanding of electronic phenomena and such knowledge is a necessary prerequisite to effective use of this manual." Within the limits so defined, the authors have produced a booklet that is at once clear, concise, and which contains a surprisingly large amount of useful information. The properties of drift tubes and reflection tubes both are dealt with, and frequency and amplitude modulation are included in the properties discussed. In this connection, the use of the term "velocity modulation" to mean velocity variation of the electron stream becomes particularly confusing, when true modulation effects are discussed in the same paragraph. The term probably has become too widespread for a change to be recommended now, but it represents a departure from the I.R.E. definition of "modulation" (3R1). It would have been helpful in the manual to have separate figures for showing the application to amplification and to oscillation. Also, the scales on the graphs in some cases are without labeling and the units in some of the equations are not named. Despite these and a few other somewhat similar omissions, those having an understanding of electronic phenomena will probably have little difficulty and will be glad to have the graphs, tables, and descriptions of operating adjustments which are included.

The manual closes with a fairly comprehensive bibliography and a list of patents bearing on the subject.

F. B. Llewellyn
Bell Telephone Laboratories, Inc.
New York, New York

Erzwungene elektrische Schwingungen an rotationssymmetrischen Leitern bei zonaler Anregung, by Ernst Metzler

Published (1943) by Verlag A. G. Gebr. Leeman and Company, Zurich. 120 pages+17 illustrations. 6½×9 inches. Price, Fr. 6.80—Rm. 4.10.

This book is primarily a research pamphlet, based on the author's own work in antenna theory. The treatment is mathematical, not too easy reading, and as a whole it is suggestive rather than exhaustive.

The problem considered is that of determining the current distribution on perfectly conducting antennas, bounded by surfaces of revolution, when excited by a uniform zonal electromotive force. The solution given is in the form of an infinite series involving the characteristic functions of the corresponding free-oscillations problem, so that the method is applicable only when the solution for free oscillations is known. The author discusses the solution for ellipsoids, spheres, and thin cylinders.

In the second part of the book the cylindrical solution is compared with some of the author's experimental work, but this part is not very satisfactory since there are no experimental points near the maximum values of the input resistance, where comparison between theory and experiment is most needed.

A fairly extensive bibliography is included, and the results of other research workers in this field are discussed throughout, but it is unfortunate that the author is not familiar with all of the work that has been done in this country in the last few years.

Marion C. Gray
Bell Telephone Laboratories
New York, N. Y.

Report of the Secretary—1943

This report has been prepared to inform the membership of the activities of the Institute during 1943.

Membership

The paid membership reached 11,079, representing a gain of 2285 members or 26 per cent for the year and a new high in Institute membership. Fig. 1 indicates the trend of the membership since 1912, the year of establishment of the Institute.

The domestic membership increased 27 per cent and the foreign membership, despite wartime restrictions, showed a gain of 18 per cent.

The proportion of the total membership outside the United States and its possessions was 10.7 per cent, as compared with 11.6 per cent in 1942.

The number of members in the British Empire rose 22.6 per cent in 1943, as compared with the increment of 13.6 per cent during 1942. The total of the 1943 European membership represents an increase of 8.5 per cent due mainly to the gain in England, whereas a loss of 4.2 per cent occurred during the previous year.

The South American membership of 120 includes a reduction of three members, or 2.4 per cent, in contrast with the total for 1942.

The distribution of the membership by grades, including that of Senior Member which had been adopted, is analyzed in Table I.

TABLE I

MEMBERSHIP DISTRIBUTION BY GRADES

		Per Cent
Fellow	185	1.7
Senior Member	886	8.0
Member	17	0.2
Associate	7,705	69.5
Student	2,286	20.6
	11,079	100.0

The total of 2745 new members elected to the Institute represents an increase of 32.9 per cent in comparison with the 2065 of the previous year.

The 2783 applications for admission and transfer, received during the year, include a gain of 40 per cent over the 1984 submitted during 1942. The number of Student applications was 21 per cent larger than that for the previous year. The applications for all other elective grades combined increased 59 per cent, in contrast with a reduction of four per cent in the previous year.

The increase in number of workers in the radio and allied fields and of those trained for radio equipment in the armed services, brought about as the result of the war, has continued to expand the field from which the Institute obtains new members.

The activities of the Membership Committee, under the direction of Chairman B. J. Thompson, and of the Admissions Committee, under Chairman G. T. Royden, necessitated in part by the adoption of the Constitutional amendments, were largely responsible for the new high in Institute membership that took place in 1943.

The Sections and many of the members accounted for a volume of applications that was substantially in excess of that of the previous year.

Proceedings

Several new features were inaugurated in 1943 including the use of pictures on the front cover, frontispieces in each issue, and guest editorials, which appear monthly. A total of 79 technical papers and 48 book reviews was included in volume 31 of the PROCEEDINGS. There were published 700 pages of technical and editorial material, as compared with 560 pages in 1942.

Because of severe paper limitations, it has been necessary to cut the weight of paper stock by almost 50 per cent. Other technical economics, which were put into practice, made it possible to save space and paper.

The Institute regrets having to reduce

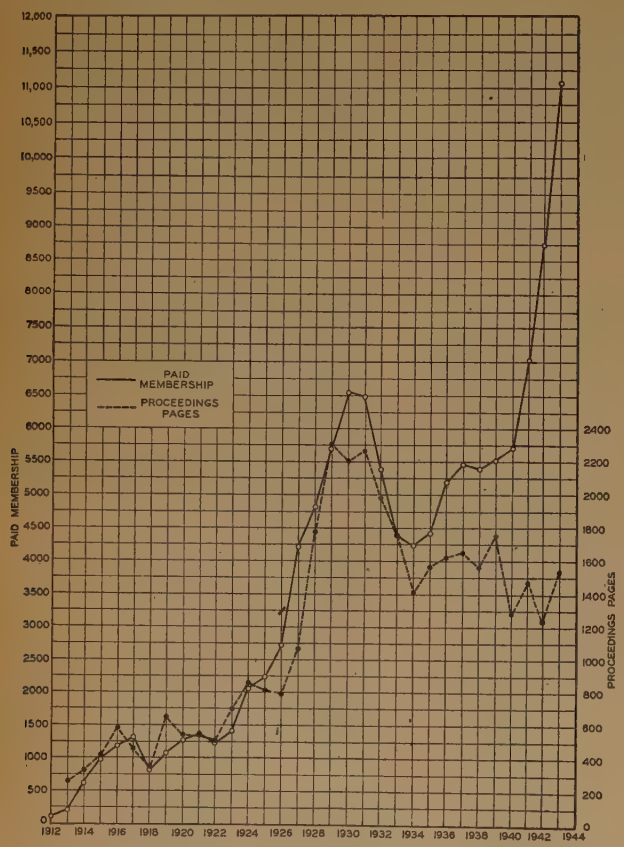


FIG. 1—The variation in paid membership is shown by the solid graph. The dotted line is for the number of pages of technical and editorial material in the PROCEEDINGS. Starting in 1939, a larger format was used and the scale of pages should be divided by 2.2.

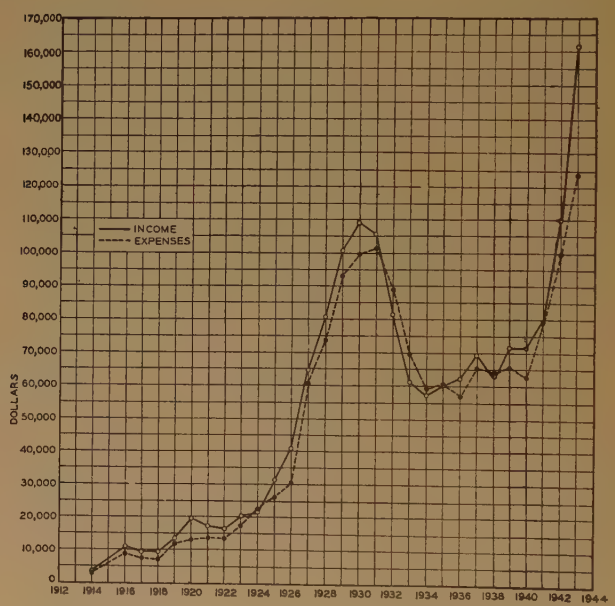


FIG. 2—Income and expenses are plotted for the life of the Institute.

the weight of the paper stock used in printing the PROCEEDINGS, and the consequent annoyance to readers caused by such condition. Numerous measures have been taken in an effort to make the PROCEEDINGS as readable as possible under the circumstances and, still, to provide sufficient copies for the steadily increasing membership.

In addition to his duties as Editor of the PROCEEDINGS, Dr. Goldsmith took on the added responsibility of becoming Managing Editor. Partly as a result of his untiring efforts in behalf of the PROCEEDINGS, the high quality of the publication was upheld and the many innovations for some of which he was directly responsible greatly enhanced its value. A wide range of added responsibilities have devolved upon the Associate Editor, Miss Helen M. Stote, and have been capably carried out. Mr. Israel, as Chairman of the Papers Procurement Committee, effectively devoted great amounts of time and effort to the PROCEEDINGS and was most successful in securing many papers for publication. Dr. Llewellyn, as Chairman of the Papers Committee, was responsible for the oustandingly successful work done by that group. The authors who submitted papers to the PROCEEDINGS were unfailingly cooperative and provided it with manuscripts of great merit. The editorial readers of the submitted papers unstintingly gave valuable time and thought to their duties. In spite of many difficulties, the printer maintained his accustomed standard of high performance.

Greater elasticity in placement was permitted between certain portions of the editorial and advertising sections of the PROCEEDINGS so that now considerable editorial material is to be found in the advertising section. Page numbers in the advertising section are now in arabic numerals followed by a letter, in place of the Roman numerals formerly used.

Sections

A total of 211 Section meetings was held by the Institute's 27 Sections during 1943. This number of meetings is 5 per cent in excess of those in 1942.

The proportion of the total membership in Section territories was reduced from 85 to 76 per cent in 1943. Numerically, the total membership in Sections rose from 6525 to 8421 at the end of 1943.

Visits of the President

President Wheeler visited the Boston, Philadelphia, and Washington Sections during 1943.

Winter Technical Meeting

The 1943 Winter Technical Meeting, held on January 28 and 29 in New York City, included the presentation of 22 technical papers. There were 808 present at the banquet. The registered attendance totaled 1704 members and guests.

Rochester Fall Meeting

The Rochester Fall Meeting took place on November 8 and 9 in Rochester, New York. There were 12 technical papers presented during the two-day session. More than 270 members and guests attended the banquet. A total of 524 were registered.

Board of Directors

The Board of Directors held 11 meetings during 1943.

Executive Committee

The Executive Committee held 12 meetings during the same period.

Administrative Committees

The 15 administrative committees, whose activities bear directly on the manifold operations of the Institute and relations with members, held 17 meetings during the year and in addition accomplished a large volume of work through the greater use of correspondence and the telephone.

Special Committees

Six special committees, appointed to carry out assignments relating to Institute activities, held 20 meetings and also utilized correspondence and the telephone in performing their duties.

Technical Committees

The meetings of the 30 technical committees totaled ten in comparison with seven in the previous year. In some cases, the committee work was partially or wholly done through the medium of correspondence.

Comparative Statement of Income and Expenses for the Years Ending December 31, 1943, and 1942

	1943	1942
INCOME		
Dues, Current and in Arrears	$ 59,866.05	$ 49,363.60
Entrance and Transfer Fees	4,818.00	3,551.00
Subscriptions	12,123.37	8,773.06
Advertising	77,858.27	38,010.00
Binders, Bound Volumes, Emblems, Reprints	4,239.32	3,643.59
Interest from Investments[1]	1,410.37	1,414.47
Conventions	—	2,970.37
Miscellaneous	717.34	2,527.48
TOTAL INCOME	$161,032.72	$110,253.57
EXPENSES		
Advertising Commissions, Salaries, Expenses	$ 26,904.20	$ 13,214.89
Bad Debts, Less Recoveries[2]	2,496.78	2,854.24
Binders, Bound Volumes, Emblems, Reprints	2,960.86	2,730.82
Conventions	365.86	4,447.60
Membership Solicitation	333.60	401.50
New York Meetings	—	866.81
Office	8,859.71	7,097.98
1943 / 1942		
Depreciation — $ 843.03 / $ 775.05		
Insurance — 227.72 / 200.14		
Postage — 3,959.85 / 2,858.56		
Stationery, Supplies — 2,926.61 / 2,417.40		
Telegraph, Telephone — 902.50 / 846.83		
Printing	36,299.54	27,697.01
PROCEEDINGS — 33,085.90 / 21,061.31		
Standards — 855.95 / 2,533.35		
Yearbook — — / 3,725.84		
Miscellaneous — 2,357.69 / 376.51		
Professional Services, Accounting, and Management	540.00	700.00
Rent and Electricity	3,880.54	3,650.32
Salaries	29,268.07	29,672.65
General — 24,266.07 / 20,541.41		
PROCEEDINGS — 5,002.00 / 4,153.78		
Standards — — / 3,259.90		
Yearbook — — / 1,717.56		
Sections	5,446.15	4,196.54
Radio Technical Planning Board	1,108.33	—
Securities Exchange Loss[3]	2,747.07	—
Miscellaneous	3,529.17	2,518.64
TOTAL EXPENSES	$124,739.88	$100,049.00
CARRIED TO SURPLUS	$ 36,292.84	$ 10,204.57

[1] Morris Liebmann Memorial Fund and Prize not included in this accounting.
[2] At least 99 per cent of this amount represents nonpayment of dues.
[3] Sale of Baltimore and Ohio Railroad bonds considered to be advisable.

Summary of Meetings

The foregoing groups held a total of 70 meetings, which are analyzed in Table II, or 11 more than in the previous year:

TABLE II

MEETINGS HELD DURING 1943

11	Board of Directors
12	Executive Committees
17	Administrative Committees
10	Technical Committees
20	Special Committees
70	

Constitution and Bylaws

During the year, the Constitution was amended by vote of the membership. The amendments included a revision of the membership structure, a broadening of the engineering field of the Institute, an increase in the period for paying dues, changing the title of "Chairman of the Board of Editors" to that of "Editor," a modification to facilitate office management, and delineating the terms of appointed officers.

Several amendments of the Bylaws were also adopted.

Awards

The Institute Medal of Honor for 1943 was presented to Dr. William Wilson for his achievements in the development of modern electronics, including its application to radiotelephony, and for his contributions to the welfare and work of the Institute.

The Morris Liebmann Memorial Prize for 1943 was given to Dr. W. L. Barrow for his theoretical and experimental investigations of ultra-high-frequency propagation in wave guides and radiation from horns, and the application of these principles to engineering practice.

In recognition of their contributions to radio, the following ten members of the Institute were transferred to the Fellow grade:

Andrew Alford	D. E. Harnett
I. S. Coggeshall	D. D. Israel
J. B. Dow	A. G. Jensen
Lee Du Bridge	G. F. Metcalf
P. C. Goldmark	Irving Wolff

Finances

The comparative statement of income and expenses for the calendar years 1943 and 1942, reproduced herewith, is taken from the auditor's reports, prepared by Klauser and Todt, Certified Public Accountants. Also, the income and expenses for each year of the life of the Institute are charted in the accompanying Fig. 2.

Headquarters Office

During the year there was considerable turnover of clerical personnel. In addition, the Cashier was granted a leave of absence due to illness.

At the close of the year the staff totaled 16 full-time employees, excluding the personnel of the advertising section employed on a contractual basis.

The adoption of an overtime policy and certain changes in methods and forms made it possible, with no increase in number of employees, to keep abreast of the increasing volume of work resulting primarily from the all-time high in Institute membership.

Deaths

The deaths of one Fellow, seven Senior Members, ten Associates, and a Student, whose names are listed below, were reported during 1943:

FELLOW

Stone, John Stone (M'13-F'15)

SENIOR MEMBER

Carter, A. J. (M'36-SM'43)
De Burgh, D. H. (A'24-M'26-SM'43)
Grimes, David (A'20-M'28-SM'43)
Hanson, M. P. (A'21-M'29-SM'43)
Knight, A. W. (A'25-M'36-SM'43)
Terrey, C. J. (A'41-M'43-SM'43)
Thompson, S. T. (A'34-M'41-SM'43)

ASSOCIATE

Angus, G. W. (A'31)
Barstow, Allan (A'41)
Brimberg, Isaac (A'36)
Grimly, E. C. (A'39)
Hard, J. M. B. (A'33)
Hodgson, Edward (A'23)
Kirkland, R. D. (A'31)
Larson, J. M. (A'42)
Thompson, Thomas (A'42)
Wadsworth, H. A. (A'35)

STUDENT

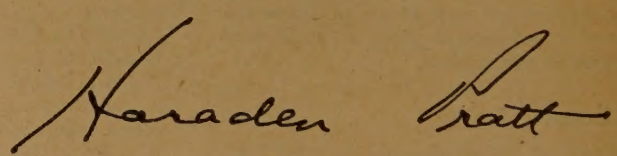
Mott, J. B. (S'42)

Acknowledgment

Further and substantial progress was made by the Institute during 1943 under the able and active leadership of President Lynde P. Wheeler. Among the year's notable accomplishments are the new all-time record in number of members, and the formation of the Radio Technical Planning Board in which the Institute played a major part.

Despite the wartime limitations, the Board of Directors, the Executive Committee, and the other committees contributed amply of their time and efforts which enabled the Institute to have one of the most successful years in its history.

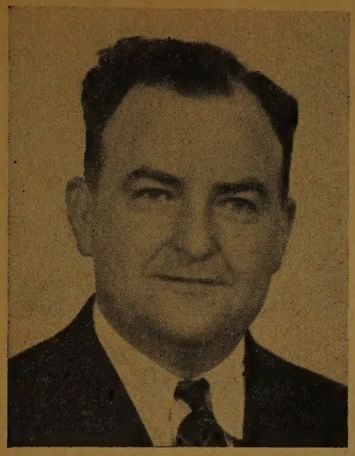
Respectfully submitted,



June 12, 1944

HARADEN PRATT
Secretary

Contributors



LEONARD J. BLACK

Leonard J. Black (A'31) was born at Santa Clara, California, on July 29, 1905. He received the B.S. degree in electrical engineering in 1928, the M.S. degree in 1930, and the Ph.D. degree in 1935 from the University of California. From 1928 to 1929 Dr. Black was with the Pacific Telephone and Telegraph Company. He was instructor in electrical engineering at the University of California from 1931 to 1940 when he became assistant professor.

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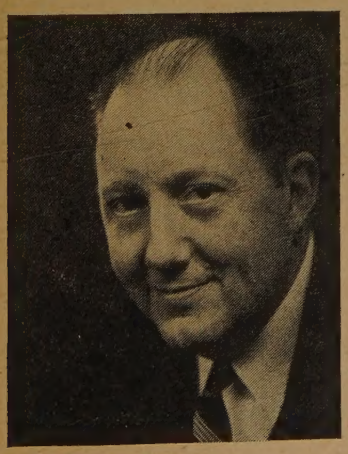
Ivan S. Coggeshall (A'26-M'29-F'43) was born at Newport, Rhode Island, on September 30, 1896, and attended Worcester Polytechnic Institute. Since 1917 he has been employed by the Western Union Telegraph Company in New York, having been appointed, in 1927, general traffic supervisor of the company's submarine cable system. Mr. Coggeshall is a Lieutenant Commander in the United States Naval Reserve, and serves on the Cable Committee of the Board of War Communications. He is a director of the Mexican Telegraph Company, a director of the Institute of Radio Engineers, and a member of Tau Beta Pi and the American Institute of Electrical Engineers.



IVAN S. COGGESHALL

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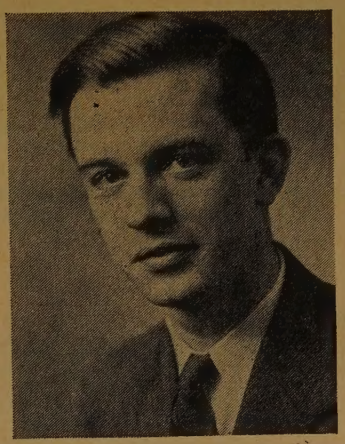
George H. Floyd (S'41–A'43) was born on May 24, 1917 at Toledo, Ohio. He received the B.S. degree in electrical engineering from the University of Arizona in 1941. Since that time he has been employed as an engineer in the tube division of the electronics department of General Electric at Schenectady, New York. Mr. Floyd is a member of Tau Beta Pi and Pi Mu Epsilon.



GEORGE H. FLOYD

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Joseph C. Hromada (J'27–A'27) was born in Chicago, Illinois, on November 4, 1906. He received the B.S. degree in electrical engineering from Armour Institute of Technology in 1929. The same year he entered the employ of the Government as a junior radio engineer in the airways division, Bureau of Lighthouses, Washington, D. C., being promoted to assistant radio engineer in 1931. He was successively associate radio engineer and radio engineer, engaged in the development of radio aids to air navigation in the Bureau of Air Commerce, which later became the Civil Aeronautics Authority. In 1939 he was transferred to Indianapolis, Indiana, as chief of the experimental station, which had just been established. He remained as chief of the station until July, 1942, when he returned to Washington, D. C., as assistant chief of the technical development division, which position he now holds.

Mr. Hromada is a member of Eta Kappa Nu. He has served on various aviation radio committees and is the author of a number of papers dealing with developments of radio aids to air navigation.

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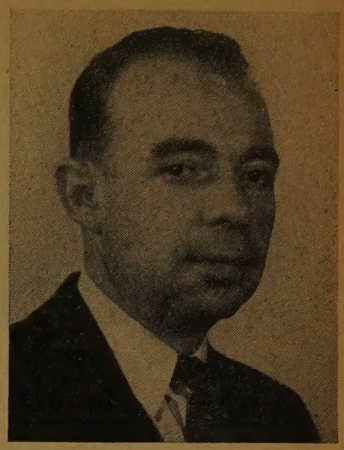
J. P. Jordan, was born in Montana in 1914. After receiving his B.S. degree in electrical engineering from the Moore School of Electrical Engineering, University of Pennsylvania, Mr. Jordan went on test at General Electric.



JOSEPH C. HROMADA

In March, 1939, he became identified with the application and design of electronic heaters in the radio transmitter engineering department.

On January 1, 1943, he was transferred to the company's industrial heating division and placed in charge of the electronic heating applications section.

Mr. Jordan is an Associate member of the American Institute of Electrical Engineers.

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Paul L. Morton (A'43) was born at Silao, Mexico, on May 14, 1906. He received the B.S. degree in electrical engineering from the University of Washington in 1931, the M.S. degree from the Massachusetts Institute of Technology in 1938, and the Ph.D. degree from the University of California in 1943. Dr. Morton was an instructor at the University of Washington from 1935 to

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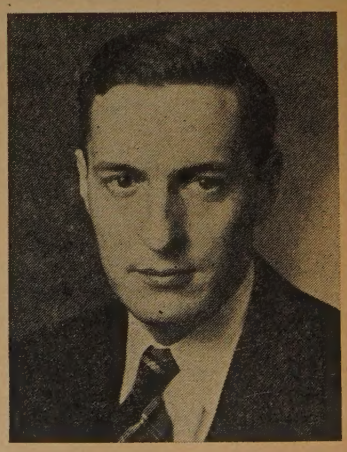
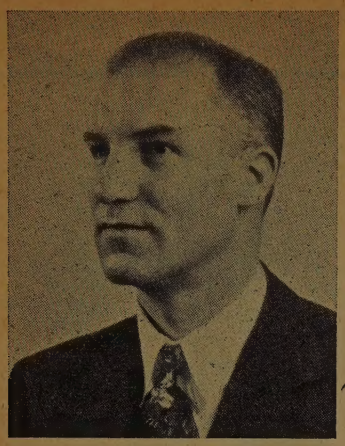


J. P. JORDAN



PAUL L. MORTON

1937, research assistant in charge of the Network Analyzer at the Massachusetts Institute of Technology from 1938 to 1939, and since 1939 has been a member of the electrical engineering staff at the University of California, becoming an assistant professor in 1943.

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Kurt Schlesinger (A'41) was born on April 20, 1906, in Berlin, Germany. In 1928 he received the engineer's diploma, and in 1929 the degree of Doctor of Applied Physics both from Technische Hochschule in Berlin. From 1929 to 1930 he was research physicist in Ardenne Research Laboratory, Berlin, and from 1931 to 1937 was chief engineer in the television department of Loewe Radio Company, also in Berlin. In 1938 Dr. Schlesinger became affiliated with the Radio and Cables-Grammont in Paris, France, where he devoted his time to television development. From 1941 to the present time he has been a research engineer for the Radio Corporation of America, attached to the laboratory at Purdue University.

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Charles L. Shackelford (S'41-A'44) was born at Wagoner, Oklahoma, on October 19, 1918. He received the B.S. degree in electrical engineering from the Oklahoma Agricultural and Mining College in 1941 and the M.S. degree from the University of Missouri in 1942. He was a teaching assistant in the electrical engineering department of the University of Missouri during 1941 and 1942. Since 1942 Mr. Shackelford has been

KURT SCHLESINGER

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with the Westinghouse Electric and Manufacturing Company. At present he is an engineer in the electronics engineering division of that company's Bloomfield, New Jersey works. Mr. Schackelford is an associate member of the American Institute of Electrical Engineers and a member of Sigma Xi, Eta Kappa Nu, and Sigma Tau.

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Bernard O. A. Thomas was born on December 14, 1912, at Hankinson, North Dakota. He is the recipient of the following

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C. L. SHACKELFORD

BERNARD O. A. THOMAS

degrees: D.D.S., 1935; B.A., 1936; and M.S. in dental surgery, all from the University of Minnesota; D.D.S., Columbia University, 1940; at present, candidate for the Ph.D. degree in Anatomy at Columbia.

From June to October, 1935, Dr. Thomas practised dentistry at Faribault, Minnesota; October, 1935, to September, 1939, he was a dental intern at the Minnesota General Hospitals, Minneapolis; October, 1936, to September, 1939, Fellow in dental surgery, Mayo Foundation, Rochester, Minnesota; 1940 to date, instructor in oral histology and operative dentistry, Columbia University, School of Dental and Oral Surgery, Presbyterian Medical Center, New York, N. Y. He is a member of the American Dental Association, New York and Minnesota; State Dental Societies, First District Dental Society of the State of New York; International Association for Dental Research; New York Academy of Sciences; American Association for the Advancement of Science; Mayo Foundation Alumni Association; Psi Omega; Omicron Kappa Upsilon; Sigma Xi; Active Fellow, New York Academy of Dentistry; American Association of Dental Editors; and Licensee, National Board of Dental Examiners. Dr. Thomas is the abstracts editor of the *Annals of Dentistry* and faculty advisor of the Dental Abstracts Society at Columbia University.

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For a biographical sketch of D. L. WAIDELICH, see the PROCEEDINGS for June, 1944.